A Stepwise-then-Orthogonal Regression (STOR) with quality control for Optimizing the RFM of High-Resolution Satellite Imagery

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A Stepwise-then-Orthogonal Regression (STOR) with quality control for Optimizing the RFM of High-Resolution Satellite Imagery

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Abstract

There are two major problems in Rational Function Model (RFM) solution: (a) Data source error, including gross error, random error, and systematic error; and (b) Model error, including over-parameterization and over-correction issues caused by unnecessary RFM parameters and exaggeration of random error in constant term of error-in-variables (EIV) model, respectively. In order to solve two major problems simultaneously, we propose a new approach named stepwise-then-orthogonal regression (STOR) with quality control. First, RFM parameters are selected by stepwise regression with gross error detection. Second, the revised orthogonal distance regression is utilized to adjust random error and address the overcorrection problem. Third, systematic error is compensated by Fourier series. The performance of conventional strategies and the proposed STOR are evaluated by control and check grids generated from SPOT5 high-resolution imagery. Compared with the least squares regression, partial least squares regression, ridge regression, and stepwise regression, the proposed STOR shows a significant improvement in accuracy.

Introduction

A satellite sensor model, which contributes to the precise georeferenced and geopositioning (Jeong *et al.*, 2015; Li *et al.*, 2014; Tong *et al.*, 2010), DEM generation (Qayyum *et al.*, 2015), and image matching (Zhang *et al.*, 2006), describes a meaningful relationship between the object space coordinates and the corresponding image coordinates. The broadly used geometric models can be roughly divided into the rigorous physical sensor model and the generalized sensor model.

A rigorous physical sensor model is used for modeling the physical imaging process of a specific satellite sensor. Since different satellite sensors with different image processing require specific physical sensor models, the rigorous physical sensor model becomes more complex and cost for user. By contrast, the generalized sensor model is a simple mathematical description of photogrammetric exploitation. The generalized sensor method usually includes grid interpolation model, rational function model (RFM), and universal real-time model. Since its successful application in Ikonos (Dial *et al.*, 2003; Fraser *et al.* 2003), QuickBird (Li *et al.* 2007, Tong, Liu and Weng 2010), SPOT (Tao *et al.* 2001), ALOS PRISM (Hashimoto, 2003), IRS-P6 (Nagasubramanian *et al.*, 2007),Ziyuan1-02C (Jiang *et al.*, 2015), ZY-3 (Wu *et al.*, 2015), GF-1(Wu *et al.*, 2016), and other high-resolution satellite imageries (HRSI), RFM has been adopted to replace physical sensor models in photogrammetric mapping and becomes a standard way for economical and fast mapping from high-resolution satellite imagery.

RFM, related the object-space (*Latitude*, *Longitude*, *Height*) coordinates to image-space (*Line*, *Sample*) coordinates, is a form of a ratio of two cubic polynomials with 78 Rational Polynomial Coefficients (RPCs). The least-squares regression is firstly employed to estimate the optimal RPCs (Grodecki *et al.*, 2003; Tao and Hu, 2001; Tong *et al*, 2010). However, owing to the strong correlation between the 78 RPCs and a limited accurate result in RPCs estimation, various developments such as the solutions, accuracy, and numerical stability of direct RFM have been achieved. Generally, for the sake of numerical accuracy, the image- and object-space coordinates are normalized to (−1, +1)(Tao and Hu, 2001). Singular value decomposition (SVD) method has been applied to solve RPCs (Fraser *et al.*, 2006; Li *et al.*, 2009), since a design matrix is likely to be close to singularity in HRSI data. In respect of ill-posed problem caused by strong correlation among 78 RPCs, several methods have been proposed to solve the ill-posed normal equation including the ridge estimation strategy, Levenberg-Marquardt algorithm, and the artificial intelligence. The ridge estimation strategy, a revised biased estimation based on the least-squares regression, is a widely used method. Combined with the L-curve method, the ridge estimation strategy can address the ill-posed equation well and obtain stratifying RPCs easily (Yuan *et al.*, 2008). In spite of the accurate RPCs obtained, the automatic determination of the optimal regularization parameter of ridge estimation is rather hard to obtain. With regard to the shortcoming of ridge estimation, a stepwise regression for ill-posed problem by removing all of the unnecessary parameters based on scatter matrix and elimination transformation strategies has been employed. With the F-statistic as an evaluation criterion, the parameter that contributes to F-statistic more would be selected the necessary parameters in RFM (Zhang *et al.*, 2012). Simultaneously, a method named the Levenberg-Marquardt algorithm has been adopted to substitute the least squares regression and solve the RPCs (Zhou *et al.*, 2012). The Levenberg-Marquardt, specialized in dealing with ill-posed problem, combines the steepest decent method and the Gauss-Newton method and inherits the global-search of gradient descent as well as the local-fast-converge of Gauss-Newton. Furthermore, another solution combined with matrix orthogonal decomposition, Levenberg-Marquardt algorithm, and compute unified device architecture high-performance computing technique has been employed (Wu and Ming,

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2016). With the development of HRSI sensors and solutions to RFM, much attention is attached to automation, such as automatic parameters selection and automatic global or local search for best RPCs values. An automatic optimal selection of RPCs based on nested regression and automatic global search for RPCs values based on genetic algorithm (GA) (Jannati *et al.*, 2015) have been presented for weeding out the redundant parameters. However, when it comes to the situation where the number of the observation less than the unknown parameters, the least-squares solution is not unique. Thus, by the virtual use of the theory of compressive sensing, the parameters selection problem can be solved by *l*1-norm-regularized least-squares (L1LS) equivalently (Long *et al.*, 2015), which makes it possible to find the unique solution of RPCs from insufficient observations efficiently and robustly.

Although various methods have been generated for more accurate RPCs, the quality of data source and the RFM itself is not reliable in some cases. Exterior and interior orientation bias in RFM cause the RFM less convincing and high geopositioning errors in practice. Exterior orientation bias (such as the orbit and attitude errors) are usually compensated by the shift, shift and drift, and the affine models (Fraser and Hanley, 2003; Li *et al.*, 2014; Tong *et al.*, 2010) or multiple physical camera model parameters (Fraser *et al.* 2005; Grodecki and Dial, 2003).

Interior orientation bias (such as lens distortion) are usually presented stably as a systematic error in RPCs estimation. Polynomial models (Wang *et al.*, 2016) and combined interior-orientation calibration method (Cao *et al.*, 2016; Jiang *et al.*, 2015) are used to compensate the interior orientation bias in RFM. Equally, it is likewise noteworthy that the model errors including the over-parameterization problem and overcorrection problem have a remarkable influence on the accuracy of RPCs. Except that methods such as ridge estimation and Levenberg-Marquardt algorithm mentioned before, a modified RFM computation based mean-variance theory is proposed to examine how much the individual RPCs contribute to RFM (Wu *et al.*, 2015), according to the consistency in the high-solution satellite imagery and airborne light detection and ranging in 3D spatial information, a matching scheme between the georeferenced airborne lidar data and high-resolution satellite images intended to control the bias of RPCs; it is verified to improve the accuracy from 18 m to about 0.58 m (Safdarinezhad *et al.*, 2017). A total least squares adjustment in partial error-in-variables model algorithm can be applied to solve the overcorrection problem caused by random error (Xu *et al.*, 2012). However, the model errors mentioned above are rarely taken into consideration simultaneously when estimating RPCs, especially the random error.

Nevertheless, the existence of data source errors in solving RPCs and model errors in RFM can cause a significant inaccuracy:

1. The quality control of data source: (a) the gross error caused by the irregular attitude and velocity of the remote sensing platform, exercises a profound influence in the reliability of RFM solutions; (b) the random error, usually with a Gaussian normal distribution, is statistical fluctuation (in either direction) in the HRSI data due to the precision imperfections of measurement devices or the atmospheric conditions; and (c) the systematic error, generally derived from (i) temperature not being standard while taping, (ii) an index error of the like the error in satellite positioning vertical circle of a theodolite or total station instrument, and (iii) use of a level rod that is not of standard length, has a significant existence in both sample and line directions.
2. The quality control of the model: (a) over-parameterization: the 78 RPCs in RFM is strongly correlated, and the over-parameterization problem caused by unnecessary RPCs leads to a less generalized model; and (b) Overcorrection: when taken the random error into consideration both in independent and response variables, the constant term will be added an extra random error mistakenly. Thus, the result will considered to be inaccurate owing to the effect of random error exaggerated in constant term.

   This paper proposes a method named stepwise-then-orthogonal regression (STOR) to solve aforementioned problems with high quality control, and contributions to the work in this research can be roughly outlined as follows:
3. Theoretically, a novel RPCs computation method based on STOR has been proposed. The accuracy, in practice, has been significantly improved in STOR when comparing to least squares regression, partial least squares regression, ridge regression, and stepwise regression in practice.
4. The data source errors have been well controlled in the course of STOR processing: (a) The gross error is detected with 3 sigma rule; (b) the random error of data source has been comprehensively considered into the procedure of the revised orthogonal distance regression; and (c) the systematic error is compensated with Fourier series. Datasets possess high reliability and availability attributes when the three type errors alleviated. Furthermore, the model errors caused by overparameterization and overcorrection problems can be solved in the processing of stepwise regression and revised orthogonal distance regression.

The remainder of this paper is organized as follows. The conventional method of RFM parameters computation is reviewed and followed by a discussion of the stepwise-then-orthogonal regression (STOR). Two SOPT-5 HRSI data sets are used for testing the new scheme and other conventional strategies of RFM. Finally, the conclusions are outlined.

Conventional Method of RFM Parameters Optimization

The RFM is the form of a ratio of two cubic polynomials related the object-space (*Latitude*, *Longitude*, *Height*) coordinates to image-space (*Sample*, *Line*) coordinates. Given the object-space coordinates (*Latitude*, *Longitude*, *Height*), where *Latitude* is geodetic latitude, *Longitude* is geodetic longitude, and *Height* is height above the ellipsoid. The latitude, longitude and height offsets and scale factor (*LAT_OFF*, *LONG_OFF*, *HEIGHT_OFF*, *LAT_SCALE*, *LONG_SCALE* and *HEIGHT_SCALE*), the calculation of image-space coordinates begins by normalizing latitude, longitude, and height as follows(Grodecki and Dial, 2003):

$$P = \frac{Latitude - LAT\_OFF}{LAT\_SCALE}$$
$$L = \frac{Longitude - LONG\_OFF}{LONG\_SCALE} . \qquad (1)$$
$$H = \frac{Height - HEIGHT\_OFF}{HEIGHT\_SCALE}$$

The normalized line and sample image-space coordinates (Y and X, respectively) are then calculated as follows:

$$\begin{cases} Y = \dfrac{Num_L(P,L,H)}{Den_L(P,L,H)} \\ X = \dfrac{Num_S(P,L,H)}{Den_S(P,L,H)} \end{cases} . \qquad (2)$$

Let us consider RFM of full rank. The four polynomials $Num_L(P,L,H)$, $Den_L(P,L,H)$, $Num_S(P,L,H)$ and $Den_S(P,L,H)$ have the following general forms, respectively:

$$
\begin{aligned}
Num_L &= (P, L, H) = a_0 + a_1 L + a_2 P + a_3 H + \ldots + a_{19} H^3 \\
Den_L &= (P, L, H) = b_0 + b_1 L + b_2 P + b_3 H + \ldots + b_{19} H^3 \\
Num_S &= (P, L, H) = c_0 + c_1 L + c_2 P + c_3 H + \ldots + c_{19} H^3 \\
Den_S &= (P, L, H) = d_0 + d_1 L + d_2 P + d_3 H + \ldots + d_{19} H^3
\end{aligned}
\tag{3}
$$

where a_i, b_i, c_i, and d_i ($I = 0, 1, 2, \ldots, 19$) are the coefficients of RFM parameters with $b_0 = 1$ and $d_0 = 1$.

Then, Equation 2 can be converted into the following linear form with n being the number of measurements:

$$
\begin{bmatrix}
1 & L_1 & \cdots & H_1^3 & -Y_1L_1 & \cdots & -Y_1H_1^3 \\
1 & L_2 & \cdots & H_2^3 & -Y_2L_2 & \cdots & -Y_2H_2^3 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
1 & L_N & \cdots & H_n^3 & -Y_nL_3 & \cdots & -Y_nH_n^3
\end{bmatrix}
\begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{18} \\ a_{19} \\ b_1 \\ b_2 \\ \vdots \\ b_{18} \\ b_{19} \end{pmatrix}
-
\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}
= \mathbf{0}\ , \tag{4}
$$

$$
\begin{bmatrix}
1 & L_1 & \cdots & H_1^3 & -X_1L_1 & \cdots & -X_1H_1^3 \\
1 & L_2 & \cdots & H_2^3 & -X_2L_2 & \cdots & -X_2H_2^3 \\
\vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\
1 & L_N & \cdots & H_n^3 & -X_nL_3 & \cdots & -X_nH_n^3
\end{bmatrix}
\begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_{18} \\ c_{19} \\ d_1 \\ d_2 \\ \vdots \\ d_{18} \\ d_{19} \end{pmatrix}
-
\begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix}
= \mathbf{0}\ . \tag{5}
$$

Equations 4 and 5 have no relationship solving their corresponding RPCs since they represent the line and sample directions of the sensor model, respectively. The two equations can be solved independently with the same strategy. Then, Equation 4 will be discussed and represented with the following matrix form:

$$\mathbf{G} \cdot \boldsymbol{\beta} = \mathbf{Y} \tag{6}$$

where

$$
\mathbf{Y} = \begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix}, \mathrm{G} = \begin{pmatrix} 1 & \mathrm{G}_{1,1} & \ldots & \mathrm{G}_{1,38} \\ 1 & \mathrm{G}_{2,1} & \ldots & \mathrm{G}_{2,38} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \mathrm{G}_{n,1} & \ldots & \mathrm{G}_{n,38} \end{pmatrix}, \ \boldsymbol{\beta} = \left(a_0 \ldots a_{19} \ \ b_1 \ldots b_{19}\right)^{\mathrm{T}}
$$

With $G_{i,j}$ (i=1,2,…, n; j=1,2,…,38) being the corresponding elements of the coefficient matrix in Equation 6.

Typically, the least squares regression is widely used for computation of RPCs. It obtains the estimation of parameters matrixby the following equations:

$$\min = \left\|\mathbf{V} = \mathbf{G}\boldsymbol{\beta} - \mathbf{Y}\right\|_{\mathrm{F}}^2, \tag{7}$$

$$\mathbf{G}^{\mathrm{T}}\mathbf{G}\ \boldsymbol{\beta}\ = \mathbf{G}^{\mathrm{T}}\mathbf{Y}, \tag{8}$$

where the $\left\|\mathbf{V} = \mathbf{G}\boldsymbol{\beta} - \mathbf{Y}\right\|_{\mathrm{F}}^2$ is the Frobenius norm of ($\mathbf{V} = \mathbf{G}\boldsymbol{\beta} - \mathbf{Y}$), namely, $\|\mathbf{V}\|_{\mathrm{F}}^2 = \sum_i \sum_j V_{i,j}^{\ 2}$, with the $V_{i,j}$ being the element of $\mathbf{V}$ at low i and column j.

However, one of the issues in the least squares is that all the 78 unnecessary and correlational RPCs need estimating (Tao and Hu, 2001). Usually the ridge estimation and partial least squares are used for alleviating ill-posed problem caused by over-parameterization when estimating RPCs. The ridge estimation adds a diagonal elements of the normal matrix in Equation 8 before the normal matrix is inversed. Similarly, partial least squares is also adopted for generalized parameters. But both of them are hard to optimize the parameters automatically.

Moreover, for more accurate RPCs, a model named error-in-variables has been adopted in surveying. In the EIV model, the coefficients matrix G as a set of observed variables is usually regarded as a matrix with random errors (Lemmerling *et al.*, 2001; Mahboub, 2012; Peiliang, 2006; Schaffrin *et al.*, 2009, Shi *et al.*, 2014; Xu *et al.*, 2012). However, the constant term has been added by random error (i.e., $\delta_{i,1}$(i=1,2,…,n)) mistakenly like Equation 9, which caused the overcorrection problem.

$$
\begin{pmatrix}
1+\delta_{1,1} & G_{1,1}+\delta_{1,2} & \cdots & G_{1,38}+\delta_{1,39} \\
1+\delta_{2,1} & G_{2,1}+\delta_{2,2} & \cdots & G_{2,38}+\delta_{2,39} \\
\vdots & \vdots & \vdots & \vdots \\
1+\delta_{n,1} & G_{n,1}+\delta_{n,2} & \cdots & G_{n,38}+\delta_{n,39}
\end{pmatrix}
\boldsymbol{\beta} = \left(\mathbf{G}+\boldsymbol{\delta}\right)\mathbf{B} = \mathbf{Y}+\boldsymbol{\varepsilon} \tag{9}
$$

where $\delta_{i,k}$ and $\boldsymbol{\varepsilon}$ are the corrections to $\mathbf{G}$ and $\mathbf{Y}$. Generally speaking, the error corrections $\delta_{i,k}$ and $\boldsymbol{\varepsilon}$ are far smaller than $\mathbf{G}$ and $\mathbf{Y}$, and according to the assumption that the random error is Gaussian distribution in error-in-variables model, $\boldsymbol{\delta}$ and $\boldsymbol{\varepsilon}$ are similarly in the form of Gaussian distribution, which means $\frac{1}{n}\sum_{i=1}^{n}\delta_{i,k} = 0$, $\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i = 0$. k = (1, …, 39), and n is the number of observed data.

A universal $\boldsymbol{\beta}$ for EIV model by minimizing the Frobenius-2 norm of the corrections of $\mathbf{G}$ and $\mathbf{Y}$ is proposed by Golub and van Loan (Golub *et al.*, 1980):

$$\min : s\left(\boldsymbol{\delta}, \boldsymbol{\varepsilon}, \boldsymbol{\beta}\right) - \left\|\left[\boldsymbol{\delta}, \boldsymbol{\varepsilon}\right]\right\|_{\mathrm{F}}^2 \tag{10}$$

From Equation 9, it is easy to realize that the random error has been mistakenly adjusted in the column of constant term, and the criterion of Equation 10 is not satisfying for a better RFM parameter estimation.

With the intention of alleviating the model errors caused by over-parameterization and overcorrection with a high quality control, in this paper, we proposed a stepwise-then-orthogonal regression method for optimizing RFM to control data source errors and solve the over-parameterization and over-correction problems simultaneously.

RFM Parameters Estimation Based on Stepwise-then-Orthogonal Model

Stepwise-then-orthogonal regression is a comprehensive method including several steps, i.e., stepwise regression based on gross error detection, orthogonal distance regression, and systematic error compensation (Figure 1). The way of solving over-parameterization and over-correction problems with a reliable data source in the course of estimating the unknown parameters $\boldsymbol{\beta}$ will be introduced in detail in the following sections.

Stepwise Regression Based on Gross Error Detection

The necessary RPCs optimization is based on stepwise regression strategy. The stepwise regression was firstly proposed by (Hair *et al.*, 2010), but will also be briefly summarized here. In the beginning, the initial number of unknown parameters is zero. When potential parameters introduced into the equation, the stepwise regression will compute the sum of squares of partial regression of each unknown parameter until the criteria of assessment has been reached. From Equation 9 we obtain an equation of a_0 as following:

$$a_0 = \bar{Y} - a_1\bar{G}_1 - \cdots - a_{19}\bar{G}_{19} - b_1\bar{G}_{20} \cdots - b_{19}\bar{G}_{38} \tag{11}$$

where $\frac{1}{n}\Sigma_{i=1}^{n}\delta_{i,k} = 0, \frac{1}{n}\Sigma_{i=1}^{n}\varepsilon_n = 0$, $\overline{G_j} = (1/n)\Sigma_{i=1}^{n}(G_{i,j} + \delta_{i,j+1})$ and $\bar{Y} = (1/n)\Sigma_{i=1}^{n}(Y_i + \varepsilon_i)$, ($k$=1,2,…,39; j=1,2,…,38).

Then, Equation 9 without a_0 can be rewritten as follows:

$$\begin{pmatrix} L_{1,1} & \cdots & L_{1,m} \\ L_{2,1} & \cdots & L_{2,m} \\ \vdots & \cdots & \vdots \\ L_{m,1} & \cdots & L_{m,m} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_{19} \\ b_1 \\ \vdots \\ b_{19} \end{pmatrix} = \begin{pmatrix} L_{1,r} \\ L_{2,r} \\ \vdots \\ L_{m,r} \end{pmatrix} \tag{12}$$

where $L_{j,p} = \sum_{i=1}^{n}(\boldsymbol{G}_{i,j} + \delta_{i,j+1} - \bar{\boldsymbol{G}}_j)(\boldsymbol{G}_{i,p} + \delta_{i,p+1} - \bar{\boldsymbol{G}}_p)$, $L_{j,r} = \sum_{i=1}^{n}(\boldsymbol{G}_{i,j} + \delta_{i,j+1} - \bar{\boldsymbol{G}}_j)(\boldsymbol{Y}_i + \varepsilon_i - \bar{Y})$, ($j$ = 1, 2, …, m; p = 1, 2, …, m); m is the num-ber of unknown parameters, and in Equation 12, it equals 38.

The (m+1) rank scatter matrix of $L_{j,p}$ can be shown as follows:

$$\mathbf{L} = \begin{bmatrix} \mathbf{G}^T\mathbf{G} & \mathbf{G}^T\mathbf{Y} \\ \mathbf{Y}^T\mathbf{G} & \mathbf{G}^T\mathbf{Y} \end{bmatrix} = \begin{pmatrix} L_{1,1} & \cdots & L_{1,m} & L_{1,m+1} \\ L_{2,1} & \cdots & L_{2,m} & L_{2,m+1} \\ \vdots & \vdots & \vdots & \vdots \\ L_{m,1} & \cdots & L_{m,m} & L_{m,m+1} \\ L_{m+1,1} & \cdots & L_{m+1,m} & L_{m+1,m+1} \end{pmatrix} \tag{13}$$

Then, we set $l_{i,j}^{(0)}$, $l_{i,m+1}^{(0)}$ and $l_{i,m+1,m+1}^{(0)}$ to represent the elements $\mathbf{G}^T\mathbf{G}$, $\mathbf{G}^T\mathbf{Y}$ and $\mathbf{Y}^T\mathbf{Y}$ in Equation 13, respectively. Every time the potential parameters are introduced into Equation 12, the sum of squares $P_j^{(t)}$ of partial regression of each unknown parameters would be computed, and whether parameters can be introduced into the final equation will be tested with F-distribution.

$$P_i^{(t)} = (l_{i,m+1}^{(t)})^2 \,/\, l_{i,i}^{(t)} \tag{14}$$

The unknown parameters will be accepted if the significance value is $F_{\text{in}}(1, n\text{-}t\text{-}2) \geq F_{\text{out}}(1, n\text{-}t\text{-}1)$, t is the number of accepted parameters, (n-t-2) is the number of degrees of freedom with t. When the parameters selection finished by stepwise regression, Equation 12 can be rewritten with $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ as follows:

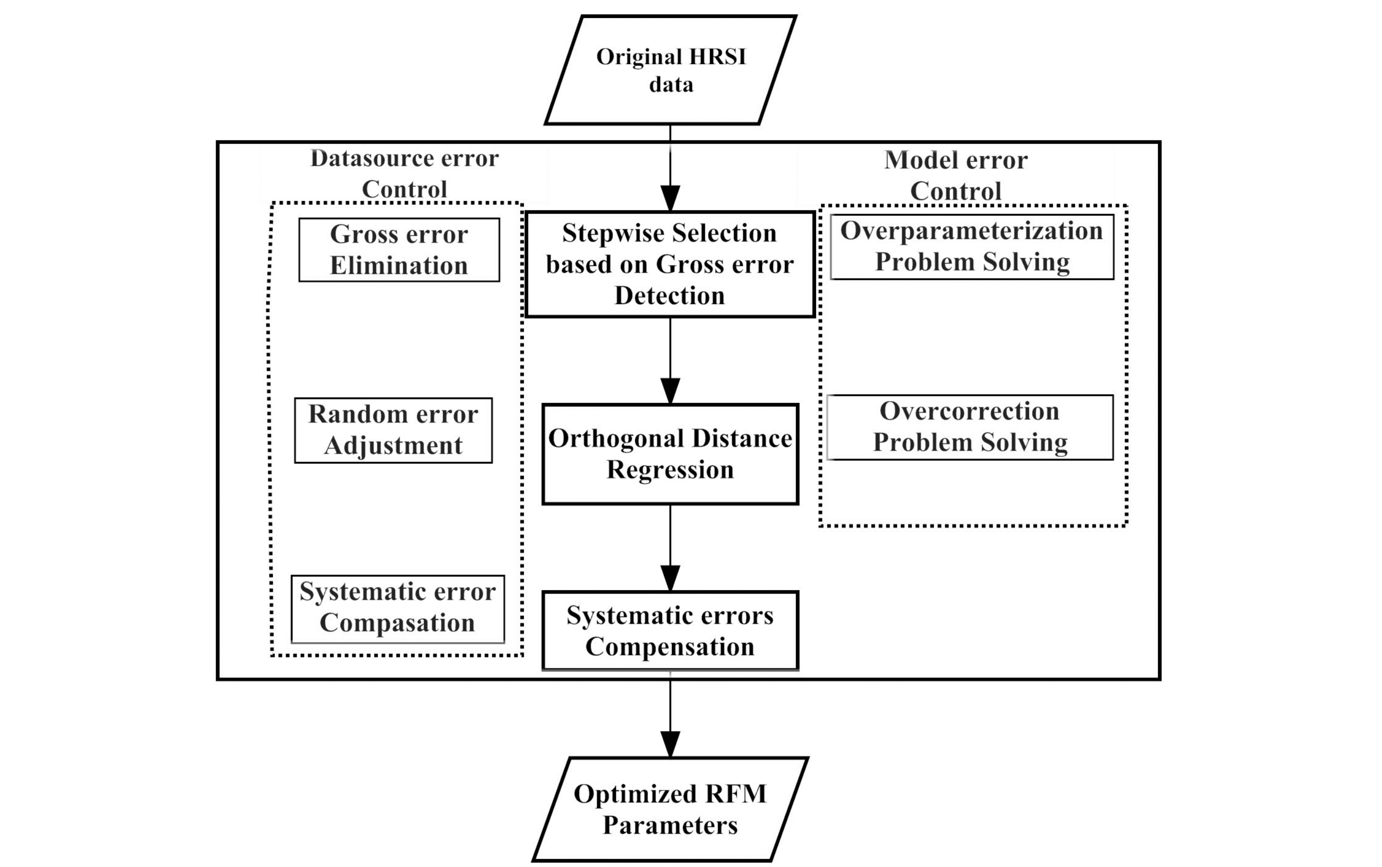


Figure 1. The procedures of STOR system for RFM parameters estimation.

$$\begin{pmatrix} L_{1,1} & \cdots & L_{1,m} \\ L_{2,1} & \cdots & L_{2,m} \\ \vdots & \cdots & \vdots \\ L_{m,1} & \cdots & L_{m,m} \end{pmatrix} (\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2) = \begin{pmatrix} L_{1,r} \\ L_{2,r} \\ \vdots \\ L_{m,r} \end{pmatrix} \tag{15}$$

where $\boldsymbol{\beta}_1$ is a meaningful vector of t accepted parameters and (m-t) zeroes, $\boldsymbol{\beta}_2$ is a vector with t zeroes and (m-t) unaccepted parameters and $\boldsymbol{\beta}' = \boldsymbol{\beta}_1 + \boldsymbol{\beta}_2 = (a_1, \ldots, a_{19}, b_1, \ldots, b_{19})^T$.

In order to make the RPCs selection more generalized, Equation 15 can be transformed into Equation 16 and the matrix form as follows:

$$\begin{pmatrix} L_{1,1} & \cdots & L_{1,m} \\ L_{2,1} & \cdots & L_{2,m} \\ \vdots & \cdots & \vdots \\ L_{m,1} & \cdots & L_{m,m} \end{pmatrix} \boldsymbol{\beta}_1 = \begin{pmatrix} L_{1,r} \\ L_{2,r} \\ \vdots \\ L_{m,r} \end{pmatrix} \tag{16}$$

$$\mathbf{L}\boldsymbol{\beta}_1 = \mathbf{L}_r \tag{17}$$

where, $\begin{pmatrix} L_{1,1} & \cdots & L_{1,m} \\ L_{2,1} & \cdots & L_{2,m} \\ \vdots & \cdots & \vdots \\ L_{m,1} & \cdots & L_{m,m} \end{pmatrix}$ and $\mathbf{L}_r = (L_{1,r}, L_{2,r}, \ldots, L_{m,r})^T$.

In stepwise regression, every time a RPC is introduced or rejected, the $\boldsymbol{\beta}_1$ will be updated and roughly estimated with the vector $\widehat{\boldsymbol{\beta}_1}$. It indicates that the RFM parameters in $\boldsymbol{\beta}_1$ are reasonable when the maximum difference (correction) in $|\Delta\boldsymbol{\beta}_1|$ of estimated $\widehat{\boldsymbol{\beta}_1}$ for two successive times is less than 10^{-6}.

$$\max\{|\Delta\boldsymbol{\beta}_{11}|, |\Delta\boldsymbol{\beta}_{12}|, \ldots, |\Delta\boldsymbol{\beta}_{1m}|\} \leq 10^{-6}. \tag{18}$$

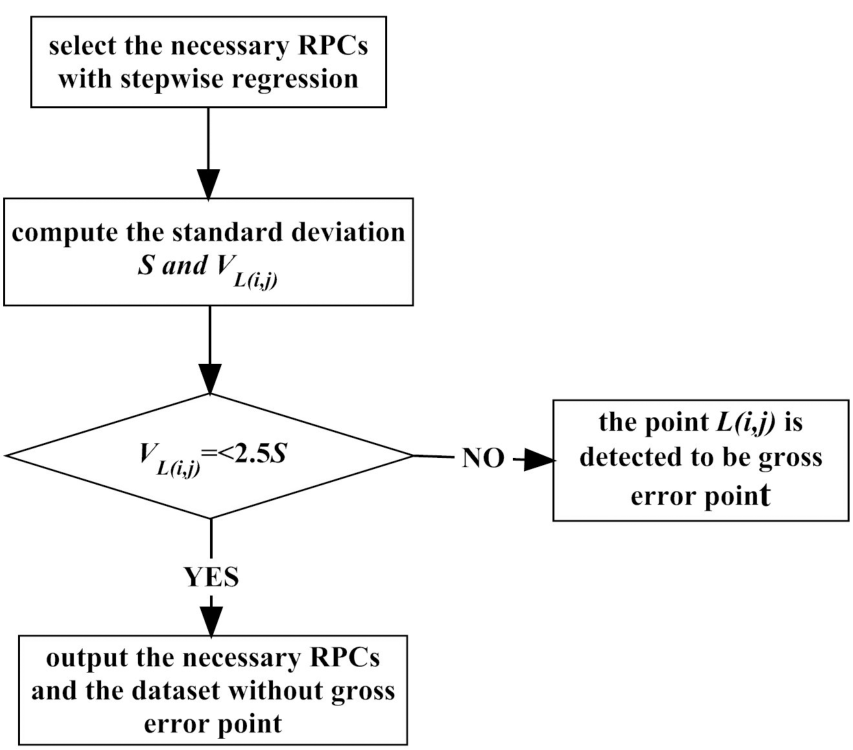


Figure 2. Stepwise regression based on gross error detection.

But for high resolution satellite imagery analysis, there is always existing an accuracy gap because of the noise in datasets. The data prediction or analysis with RFM will be more accurate when modeling as much signal as possible and as little noise as possible. Thus, in order to control the data source errors and insure a high reliable and available experiment dataset, the gross error in the data source should to be detected with the stepwise regression processing at the same time.

Gross error detection is the fundamental procedure of the parameters estimation. The combination of sum of squared residuals $\|\mathbf{V}_L\|_F^2$ and the standard deviation S obtained in stepwise regression processing is a one of criterions of iteration in gross error detection.

$$\|\mathbf{V}_L\|_F^2 = \|\mathbf{L}\boldsymbol{\beta}_1 - \mathbf{L}_r\|_F^2 = \sum_i V_{Li}^2 \tag{19}$$

$$S = \sqrt{\frac{\|\mathbf{V}_L\|_F^2}{n-t}} \tag{20}$$

where, the $\|\mathbf{V}_L\|_F^2$ is the Frobenius norm of $\mathbf{V}_L$, and the $V_{L(i,j)}$ is the element of $\mathbf{V}_L$ at low i and column j, the n and t represent the number of data points and the necessary observation number of RPCs accepted by stepwise regression, respectively.

Thus, the criterion in gross error detection with a combination of V_{Li} and S shows in Equation 21, which is performed with variable weights, generally, 3 sigma rule, the point will be detected if the residual of a point is larger than 2.5 times of the standard deviation S.

$$V_{L(ij)} \leq 2.5S \tag{21}$$

$$\gamma = [a_0 \boldsymbol{\beta}_1^T]^T \tag{22}$$

With the procedure of gross error detection, the necessary RFM parameters $\boldsymbol{\gamma}$ with a more reliable and available original dataset can be obtained, and the flow of stepwise regression based on gross error detection can be briefly summarized as the Figure 2.

Notwithstanding the optimized RPCs in $\boldsymbol{\gamma}$ selected by stepwise regression, the problem regarding how to estimate the optimized parameterswith the least impact of random error still remains for discussion. The principle of least squares regression involves the assumption that the independent variables is well-known with no error dependence. Therefore, to some extent, the existence of random errors in the measurement and determination variables disturbs the accuracy of RPCs estimation.

Orthogonal Distance Regression

Orthogonal distance regression (ODR) is the name given to the computational problem associated with finding the maximum likelihood estimators of parameters in observational error models in the case of normally distributed errors. Different from the least squares regression, the ODR can sufficiently reduce the impact of random error in independent and determination variables with the criterion of $\min : s(\delta, \varepsilon, \gamma) = \|[\delta, \varepsilon]\|_F^2$, and the criterion can be intuitively explained with Figure 3.

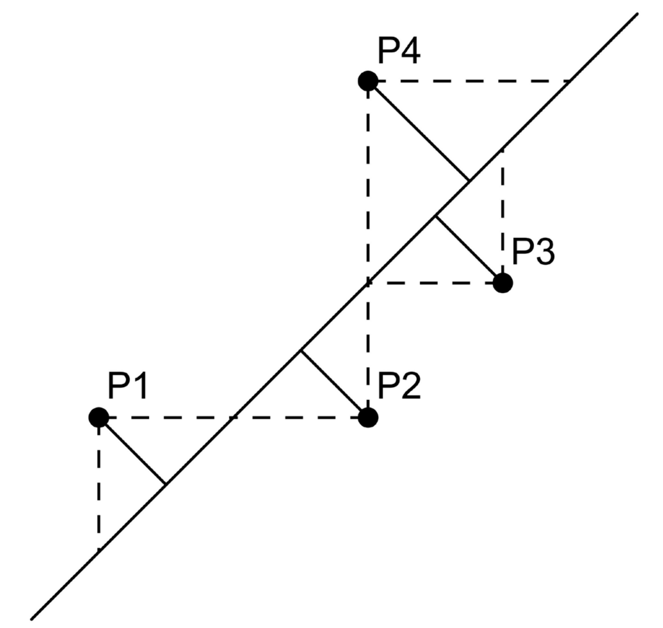


Figure 3. Geometrical illustrations of Orthogonal Distance solution: the dotted lines are the observational errors in observed variables. The solid lines represent the minimum Euclidean distance from data points to fitting the hyperplane.

In order to have a stable estimation of $\boldsymbol{\gamma}$ with singular value decomposition (SVD), the augmented matrix in Equation 9 without constant term can be rewritten as follows:

$$\mathbf{M} = \begin{pmatrix} G_{1,1}+\delta_{1,2} & G_{1,2}+\delta_{1,3} & \cdots & G_{1,t}+\delta_{1,t+1} & Y_1+\varepsilon_1 \\ G_{2,1}+\delta_{2,2} & G_{2,2}+\delta_{2,3} & \cdots & G_{2,t}+\delta_{2,t+1} & Y_2+\varepsilon_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{n,1}+\delta_{n,2} & G_{n,2}+\delta_{n,3} & \cdots & G_{n,t}+\delta_{n,t+1} & Y_n+\varepsilon_n \end{pmatrix} \quad (23)$$

$$\bar{\mathbf{M}} = \begin{pmatrix} \bar{G}_1 & \bar{G}_2 & \cdots & \bar{G}_t & \bar{\mathrm{Y}} \end{pmatrix} \quad (24)$$

when $\frac{1}{n}\Sigma_{i=1}^{n}\delta_{i,k} = 0$, $\frac{1}{n}\Sigma_{i=1}^{n}\varepsilon_i = 0$, $\bar{G}_1 = \left(\frac{1}{n}\right)\Sigma_{i=1}^{n}G_{i,1}$, …, $\bar{G}_t = \frac{1}{n}\Sigma_{i=1}^{n}G_{i,t}$, $\bar{Y} = \frac{1}{n}\Sigma_{i=1}^{n}$, $(k = 1, \ldots, t)$, Y_i, and t is the number of necessary RFM parameters selected by stepwise regression.

Considering the random error compensation both in *Line* and *Sample*, all the measurements can be treated far more precise by subtracting the mean value of each column measurements in **M,** and the matrix **M** will be treated as **Q**:

$$\mathbf{Q} = \mathbf{M} - \bar{\mathbf{M}} = \begin{pmatrix} G_{1,1}+\delta_{1,2}-\bar{G}_1 & G_{1,2}+\delta_{1,3}-\bar{G}_1 & \cdots & G_{1,t}+\delta_{1,t+1}-\bar{G}_t & Y_1+\varepsilon_1-\bar{Y} \\ G_{2,1}+\delta_{2,2}-\bar{G}_1 & G_{2,2}+\delta_{2,3}-\bar{G}_2 & \cdots & G_{2,t}+\delta_{2,t+1}-\bar{G}_t & Y_2+\varepsilon_2-\bar{Y} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{n,1}+\delta_{n,2}-\bar{G}_1 & G_{n,2}+\delta_{n,3}-\bar{G}_2 & \cdots & G_{n,t}+\delta_{n,t+1}-\bar{G}_t & Y_n+\varepsilon_n-\bar{Y} \end{pmatrix}. \quad (25)$$

To obtain the RFM parameter values of $\boldsymbol{\gamma}$, the method of singular value decomposition (SVD) is adopted. The singular value decomposition is a well-known matrix factorization technique that factors n by $(t+1)$ matrix **Q** into three matrices as follows:

$$\mathbf{Q} = \mathbf{USV}^{\mathrm{T}} = \begin{pmatrix} u_{1,1} & \cdots & u_{1,r} \\ \vdots & \vdots & \vdots \\ u_{n,1} & \cdots & u_{n,r} \end{pmatrix}_{n\times r} \begin{pmatrix} s_{1,1} & \cdots & s_{1,r} \\ \vdots & \vdots & \vdots \\ s_{r,1} & \cdots & s_{r,r} \end{pmatrix}_{r\times r} \begin{pmatrix} v_{1,1} & \cdots & v_{1,(t+1)} \\ \vdots & \vdots & \vdots \\ v_{r,1} & \cdots & v_{r,(t+1)} \end{pmatrix}_{r\times(t+1)} \quad (26)$$

where **U** is an orthogonal matrix of **Q**. **S** is a diagonal matrix containing the singular values of the matrix **Q**. There are exactly r singular values, where r is the rank of **Q**. The columns of **V** are the singular vectors of **M**. When assumed that $V_r = (V_{1,r}, V_{2,r}, \ldots, V_{t+1,r})^{\mathrm{T}}$ and $\mathbf{V}_{tr} = (V_{1,r}, V_{2,r}, \ldots, V_{t,r})^{\mathrm{T}}$ are part of elements in **V**, the RPCs values $\boldsymbol{\gamma}$, which a_0 and $\boldsymbol{\beta}_1$ can be estimated by as follows:

$$\boldsymbol{\beta}_1 = \mathbf{V}_{tr} \,/\, V_{t+1,r} \quad (27)$$

$$\boldsymbol{\alpha} = \mathbf{Y} - \mathbf{M}'\boldsymbol{\beta}_1 \quad (28)$$

where, $\mathbf{M}' = \begin{pmatrix} G_{1,1} & G_{1,2} & \cdots & G_{1,t} \\ G_{2,1} & G_{2,2} & \cdots & G_{2,t} \\ \vdots & \vdots & \vdots & \vdots \\ G_{n,1} & G_{n,2} & \cdots & G_{n,t} \end{pmatrix}$, $Y = (Y_1, Y_2, \ldots, Y_n)^{\mathrm{T}}$ and the t is number of necessary RPCs selected by stepwise regression:

$$a_0 = (1/n)\sum_{i=1}^{n}\boldsymbol{a}_i \,. \quad (29)$$

The final estimations of RPCs $\boldsymbol{\gamma}$ can be rewritten as follows:

$$\boldsymbol{\gamma} = \left[(1/n)\sum_{i=1}^{n}\boldsymbol{a}_i, \left(-\mathbf{v}_{tr} \,/\, v_{t+1,r}\right)^{\mathrm{T}}\right]^{\mathrm{T}}. \quad (30)$$

The orthogonal distance regression processing has taken the random error into account in both independent and dependent variables and successfully addressed the overcorrection problem caused by constant term. Through subtracting the mean of each column measurements from augmented matrix in Equation 19, the random error has been adjusted properly. Thus, the reliability and the accuracy of the RFM model would be significantly enhanced.

Systematic Errors Compensation

The systematic error compensation is used for alleviating the residual systematic error of RFM and improving the accuracy of RPCs. Usually, the distribution of residues shows a wavy change. Contrary to other fitting methods in experiments, the Fourier series has a more advantageous result. The Fourier series fitting model is shown as follows:

$$\begin{cases} Y + \Delta Y = \dfrac{NumL(P,L,H)}{DenL(P,L,H)} \\ X + \Delta X = \dfrac{NumS(P,L,H)}{DenS(P,L,H)} \end{cases} \quad (31)$$

where,

$$\begin{cases} \Delta Y = p_{r0} + p_{r1}\cos(w_r Y) + q_{r1}\sin(w_r Y) + p_{r2}\cos(2w_r Y) + q_{r2}\sin(2w_r Y) \\ \quad + p_{r3}\cos(3w_r Y) + q_{r3}\sin(3w_r Y) + \ldots + p_{rl}\cos(lw_r Y) + q_{rl}\sin(lw_r Y) \\ \Delta X = p_{c0} + p_{c1}\cos(w_c X) + q_{c1}\sin(w_c X) + p_{c2}\cos(2w_c X) + q_{c2}\sin(2w_c X) \\ \quad + p_{c3}\cos(3w_c X) + q_{c3}\sin(3w_c X) + \ldots + p_{ck}\cos(kw_c X) + q_{ck}\sin(kw_c X) \end{cases} \quad (32)$$

where, p_{r0}, p_{r1}, q_{r1}, …, q_{rn}, w_r, p_{c0}, p_{c1}, q_{c1}, …, q_{cn} and w_c are the Fourier series fitting coefficients l, and k is the number of fitting terms. In experiments, the values of l and k are set to be 6 and 8, respectively, which can sufficiently alleviate the systematic error.

Shown as the Figure 1 and the description of STOR algorithm, the model errors caused by over parameterization and over correction problems has been addressed by stepwise regression and orthogonal distance regression, respectively. The gross error, random error, and systematic error from data source are well controlled by the procedures of gross error detection, orthogonal distance and systematic error compensation. Through the procedures of STOR scheme, the necessary RPCs have been successfully selected and estimated with a good quality control. The intention to estimate RFM parameters with reliable data source and model have been attained.

Experiments

To test the stability and accuracy of the STOR method, two experiments were performed with spatial grids generated by SPOT5 HRS data. The introduction of the two datasets and the experimental results will be shown in detail in the following sections.

Test Datasets

The datasets, which contain the virtual control points and virtual check points, are generated by the rigorous model of SPOT5 HRS stereo images and all the original image sizes are 12,000 × 12,000 pixels. The elevation of the spatial grids varies from 200 to 2,200 m. There are in total five layers with 500 m height interval for control and check points. As shown in Figure 4. There are 552 image points evenly distributed in every image plane. The even points are used for control points, and the odd are checkpoints. A spatial ray can be determined for image point by the projection center and its image coordinate. The corresponding spatial coordinate of an image point can be calculated by intersection between the ray and a level plane with known elevation.

In this experiment, the two datasets have a procedure of gross error detection based on 3 sigma rule. If a point with a RMSE larger than 2.5 times of the standard deviation S would not be accepted and vice versa.

Results of STOR Method and Conventional Strategies

In order to select, estimate, and optimize the necessary RPCs with a good control of model errors and data source errors, we have proposed the stepwise-then-orthogonal regression (STOR) method, and make a comparison with conventional strategies in the numbers of optimized RPCs and the accuracy with different methods.

The number of optimized RPCs of different methods have been shown in Table 1. As the table shows, the proposed STOR method performed the best result in RPCs selection. In the first experimental dataset, the STOR scheme can estimate the RFM with the RPCs in sample and line are both 17, and in the second dataset shows that the numbers in sample and line directions are 23 and19, respectively. The least squares solve the RFM with 39 RPCs both in sample and line directions in two datasets. The number of optimized RPCs in partial least squares and ridge regression is 25 in sample and line directions in the two experimental datasets. The stepwise regression method can cope with this RFM estimation with 17 RPCs both in sample and line directions in the first experimental dataset, and 23 and 19 RPCs in the second experimental dataset in sample and line directions, respectively. Compared with least squares regression, the methods of partial least squares and ridge regression show a better stability, but the automatic determination of the optimized RPCs in the methods

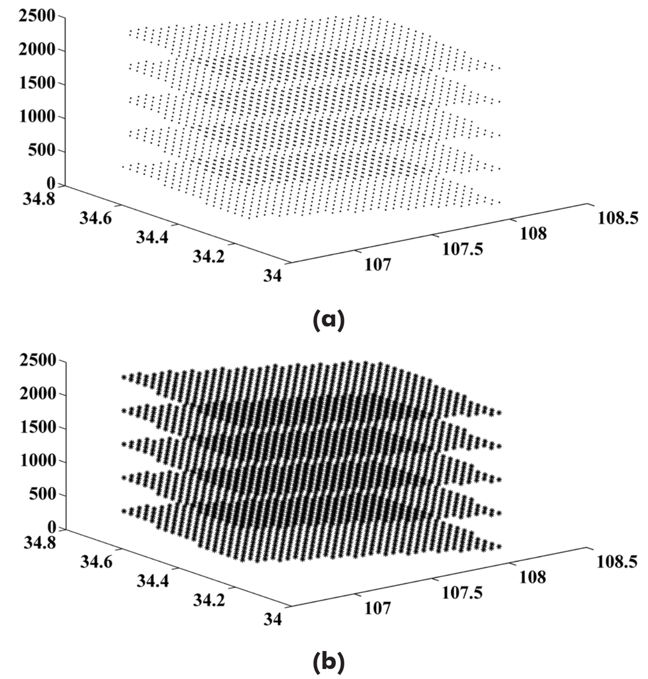


Figure 4. Spatial grids of SPOT-5 stereo image: (a) The first dataset of SPOT-5 stereo image, and (b) The second dataset of SPOT-5 stereo image.

Table 1. The Number of RPCs of different strategies.

Strategies of RPCs estimation	The number of optimized RPCs			
	The first dataset		The second dataset	
	Sample	Line	Sample	Line
Least squares regression	39	39	39	39
Partial Least Squares regression	25	25	25	25
Ridge regression	25	25	25	25
Stepwise regression	17	17	23	19
STOR	17	17	23	19

of partial least squares and ridge regression are hard to obtain in experiments. Simultaneously, the numbers of gross error points detected in the procedure of stepwise regression based on gross error are 73 and 70, respectively.

The distribution of systematic error in sample and line directions has been shown in Figure 5. Generally, the systematic error is a wavy change, which indicates that residual caused by systematic error can be well compensated. In the experiments, we found that the systematic error compensation with Fourier series emphasized a satisfying result.

1. The Fourier series fitting in Sample direction in first dataset;
2. The Fourier series fitting in Line direction in first dataset;
3. The Fourier series fitting in Sample direction in second dataset; and
4. The Fourier series fitting in Line direction in second dataset.

The accuracy of the calculated RPCs directly influences the possible applications of HRSI. In order to evaluate whether the accuracy of STOR scheme is superior to that estimated by conventional methods, the RMSEs of the calculated RPCs for the STOR scheme and conventional strategies are compared to each other. As the results shown in Table 2 and Table 3, in the first experimental dataset, the RMSEs of STOR scheme are

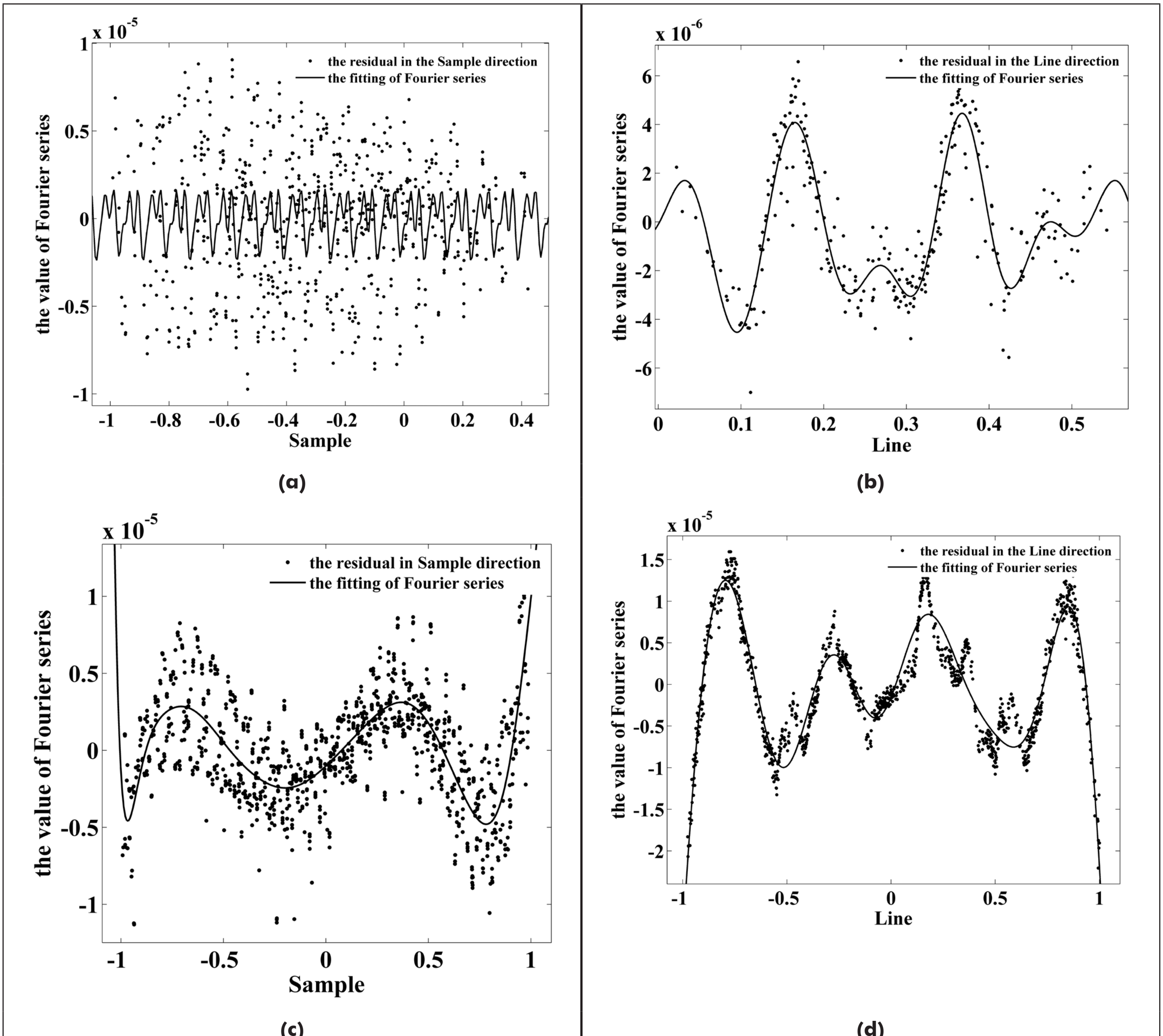


Figure 5. The systematic error compensation with Fourier series: the dots represent the residual after the stepwise-then-orthogonal regression, the line is the fitting of Fourier series, and the horizontal axis "Sample" and "Line" means the regularized image coordinate, the vertical axis shows the value of Fourier series: (a) The Fourier series fitting in Sample direction in first dataset; (b) The Fourier series fitting in Line direction in first dataset; (c) The Fourier series fitting in Sample direction in second dataset; and (d) The Fourier series fitting in Line direction in second dataset.

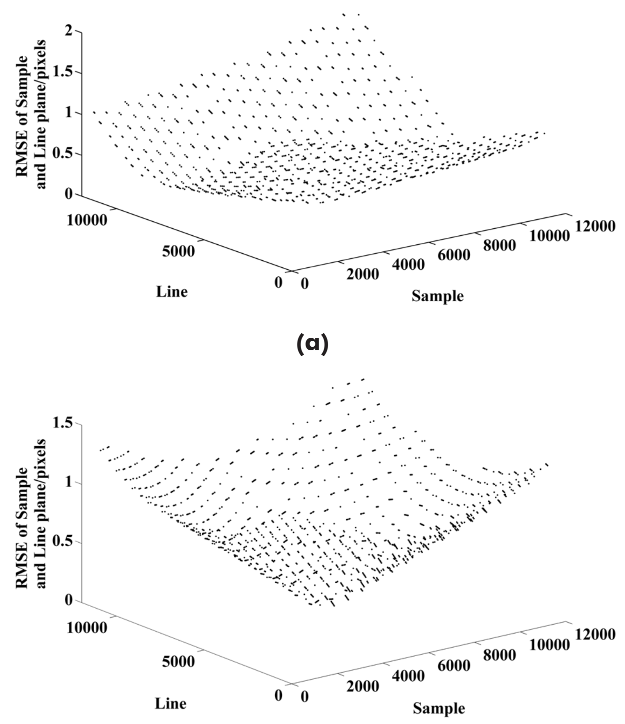
Table 2. RMSE of RFM Computation with the first dataset (Pixels).

Statistics items	Sample	Line	plane
Least Squares regression	0.01651020	0.00518250	0.01730448
Partial Least Squares regression	0.01439075	0.00686784	0.01594556
Ridge regression	0.01449994	0.00724664	0.01620993
Stepwise regression	0.01482885	0.00688105	0.01634759
STOR	0.01436922	0.00607054	0.01559891

Table 3. RMSE of RFM computation with the second dataset (pixels).

Statistics items	Sample	Line	plane
Least Squares regression	0.00789303	0.01016222	0.01286742
Partial Least Squares regression	0.00748515	0.01045084	0.01285486
Ridge regression	0.00756336	0.01082874	0.01320857
Stepwise regression	0.00789611	0.01047145	0.01311487
STOR	0.00749188	0.00688172	0.01017282

0.01436922 and 0.00607054 in sample and line, respectively. In the second dataset, the RMSEs of STOR are 0.00749188 and 0.00688172 in sample and line, respectively. And in the two experimental datasets the accuracies in sample and line plane reaches 0.01559891 and 0.01017282 pixel, which is about 3 percent higher than stepwise regression, ridge regression and partial least squares regression, and 10 percent to 20 percent higher than least squares regression and orthogonal distance regression. Thus, the accuracy of STOR is significantly higher than conventional methods. As Figure 6 shows, the RMSEs in image plane of every point in different spatial grids are centralizing around zero.



(a)

(b)

Figure 6. The RMSE distributions of STOR in SPOT-5 stereo images: (a) RMSE distribution in the first dataset, and (b) RMSE distribution in the second dataset.

Conclusion and Discussion

For RPCs selection and estimation, this paper has proposed a stepwise-then-orthogonal regression (STOR), which can address the over parameterization and over correction problems with highly reliable and available datasets. This STOR contains the procedures of stepwise regression based on gross error detection, orthogonal distance regression, and the systematic error compensation. The contributions of this novel proposed method can be roughly listed in two aspects:

1. The quality control of data source errors: (a) the gross error produced by a faulty procedure adopted can be detected with 3 sigma rule; (b) the random error caused by the precision limitation of the measurement instruments or environmental conditions in data source has be comprehensively considered into the orthogonal distance regression; and (c) the systematic error is reproducible inaccuracy in the sample or line direction. With the method of Fourier series, this kind of error has been compensated significantly.
2. The quality control of the model errors: (a) over parameterization: the necessary RFM parameters selected by the stepwise regression has improved the availability of the model and avoided the ill-posed problem to a great extent; and (b) over correction: with the random error in prediction and response variables considered into the orthogonal distance regression; the constant terms have been added a random error mistakenly. In order to estimate the constant term more scientifically, the prediction and response added a random error have subtracted the mean of the random error in each column in the processing of orthogonal distance regression.

Furthermore, as the experimental results show, the number of RPCs in STOR is less ill-posed, and the accuracy of STOR is higher 10percent to 20percent than the least squares, 3percent higher than partial least squares, ridge regression, and stepwise regression. Thus, this proposed approach can select and estimate the necessary RFM parameters with a good control of model errors and data source errors, which means that the problems of over parameterization and overcorrection can be addressed simultaneously and the reliability and availability of data source can be enhanced remarkably with the procedures of gross error detection, random error adjustment, and systematic error compensation.

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