

Approximate Correction of Length Distortion for Direct Georeferencing in Map Projection Frame

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Abstract—Many geometric distortions, such as earth curvature distortion and length distortion, exist in the map projection frame. Therefore, in aerial photogrammetry, if direct georeferencing is performed in the map projection frame, the crucial work becomes compensating for the effect of the various geometric distortions. This letter mainly focuses on length distortion and proposes two new correction approaches: the changing image coordinates method and the changing object coordinates method. The experimental results show that the changing object coordinates method is less influenced by terrain fluctuation, and its correction accuracy is therefore commonly higher than the changing image coordinates method as well as two existing approaches (i.e., the changing flight height method and the changing focal length method).

Index Terms—Distortion, georeferencing, map projection, scene restitution, sensor orientation.

I. INTRODUCTION

IN the field of aerial photogrammetry, georeferencing is commonly composed of two major consecutive procedures: sensor orientation and scene restitution [1]. The objective of the first process is to accurately acquire the exterior orientation parameters (EOPs) of the imaging sensor, and the second one computes the ground coordinates by using sensor orientation results and imaging sensor observations. Since its introduction to the area of aerial photogrammetry at the end of the last century, integrated GPS/INS navigation technology has significantly changed the data processing flow of georeferencing [2]. If the lever arms and the boresight angles between the airborne camera, GPS, and inertial measurement unit (IMU) are carefully calibrated, the position and orientation system (POS) is capable of accurately measuring the EOPs of the imaging sensor, which can be directly used in scene restitution and other subsequent process (e.g., the generation of digital terrain models and orthoimages). Therefore, this method is commonly referred to as direct georeferencing (DG). As DG does not need ground control points (GCPs) and bundle adjustment, the field survey and indoor work can be significantly reduced. However, due to the lack of constraints by the GCPs and tie points (TPs),

the ground coordinates computed by direct georeferencing are susceptible to miscellaneous systematic errors.

The data products of aerial photogrammetry are commonly required in national coordinates (i.e., using national geodetic datums and national map projections), and it is therefore reasonable to directly use the national frames in the data processing of georeferencing [3]. However, the national coordinates are usually defined by the form of the conformal map projection, which is not a Cartesian-type reference frame; therefore, scene restitution is inevitably influenced by the various map projection distortions of the form used. In the case of DG, all geometric distortions directly propagate into the ground coordinates because the scene restitution has the nature of extrapolation [4].

Of all the possible map projection distortion factors, earth curvature distortion and length distortion have been proven to lower the computational accuracy of DG dramatically. The research on earth curvature correction has a long history of at least 60 years. There are some mature solutions already available in a great deal of the literature [5], [6]. However, length distortion began to receive attention only recently because it least affects the accuracy of the traditional indirect georeferencing approach. As far as we know, there are only two existing length correction approaches for DG in the map projection frame (i.e., the changing flight height method [1], [3], [7] and the changing focal length method [4], [7], [8]). Both of these algorithms can effectively eliminate length distortion, which has been verified by many researchers using simulated or real data. However, both algorithms use the average ground elevation to calculate the length corrections. Therefore, if the real ground points deviate from the average ground elevation plane (i.e., terrain fluctuation), some non-negligible height errors can exist in the restitution results [3].

In this letter, we will present formulas for changing flight height method and changing focal length method in detail; and we further propose two new correction approaches: changing image coordinates method and the changing object coordinates method. The simulated experiment shows that, in real data scenarios, the correction accuracy of the changing object coordinates method can be significantly higher than the other three methods because it is less influenced by terrain fluctuation. The remaining sections of this letter are organized as follows. First, we briefly introduce the imaging geometry of vertical photographs and the traditional earth curvature correction method and analyze the geometric characters of length distortion in detail. Then, in Section III, the formulas of the four different length correction methods are derived. Finally, we present and

Manuscript received November 26, 2012; revised January 22, 2013 and March 18, 2013; accepted April 17, 2013. This work was supported in part by the National Basic Research Program of China under Grant 2012CB719904, the National Natural Science Foundation of China under Grant 41171292.

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Digital Object Identifier 10.1109/LGRS.2013.2259463

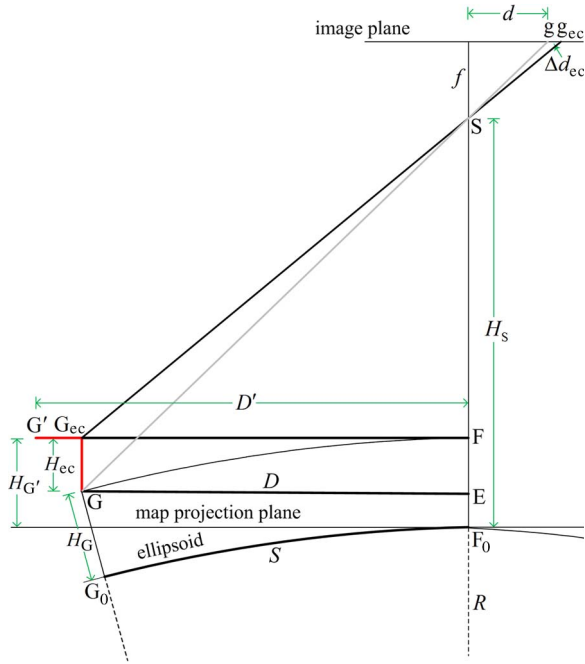


Fig. 1. Geometric distortions of the map projection frame and earth curvature correction. The gray line gG and the black line $g_{ec}G_{ec}$ represent the original light ray and the light ray after the earth curvature correction, respectively.

analyze our experimental results in Section IV and offer our concluding remarks in Section V.

II. MAP PROJECTION FRAME

A. Imaging Geometry

Like most past studies [1], [3], [4], [7], this letter only considers the imaging geometry of vertical photographs. As shown in Fig. 1, if we ignore all random and systematic errors (e.g., atmospheric refraction), then in the Cartesian coordinate system, the perspective center S , the ground point G and its corresponding image point g will lie in a straight line.

After map projection, the ground point G will become the projected ground point G' , and the collinearity condition with the perspective center S and the image point g will be destroyed. Apparently, if we still use the original parameters (i.e., the radial distance of the image coordinates d , the focal length f , and the flight height H_s) to restitute the scene, the computational result will inevitably deviate from the correct position (i.e., the projected ground point G'). The conventional solution is to alter certain computational parameters (e.g., image coordinates, flight height) in the process of scene restitution, which aims to “move” the light ray to point to the projected ground point G' , and, consequently, re-establish the collinearity condition.

B. Earth Curvature Distortion and Correction

Earth curvature distortion H_{ec} is the height difference between the ground point G and the projection ground point G' . As shown in Fig. 1, the traditional earth curvature correction

method changes the image coordinates to move point G toward point G_{ec} ; therefore, the following equation exists:

$$\frac{d_{ec}}{D} = \frac{d + \Delta d_{ec}}{D} = \frac{f}{H_s - H_{G'}} = \frac{f}{H_s - H_G} \quad (1)$$

where H_G and $H_{G'}$ are the ellipsoidal height of point G and the projection elevation of point G' , respectively, which are the same value; Δd_{ec} is the radial correction of the image coordinates, which could be calculated from the earth curvature distortion H_{ec} (cf. Fig. 1).

C. Length Distortion

Length distortion is the coordinate difference between the Cartesian length D and the projected length D' . The calculation process of D' usually requires two consecutive procedures. The first step converts the space line GE into the geodesic line $\widehat{G_0F_0}$. Note that in aerial photogrammetry, the geodesic distance S is always far shorter than earth radius R ; the geodesic line $\widehat{G_0F_0}$ therefore could be approximated by the circular arc $\widehat{G_0F_0}$. Then, according to the geometric relations in Fig. 1, we could obtain

$$S = R \arcsin \frac{D}{R + H_G} \approx \frac{RD}{R + H_G}. \quad (2)$$

In the second step, the geodesic line $\widehat{G_0F_0}$ is converted to the projection line FG' . The specific formula is determined by the map projection method used. If transverse Mercator projection [9] is chosen, then

$$D' = KS \quad (3)$$

where K is line scale factor [9], [10]. The calculation process of K is quite complicated because it involves the projected ground coordinates [9]; therefore, the approximate solution is to use the point scale factor k instead of K . Further, substituting (2) into (3), it yields

$$D' \approx \frac{kR}{R + H_G} D. \quad (4)$$

The point scale factor k is usually calculated by [11]

$$k = k_0 \left(1 + \frac{X_S^2}{2k_0^2 R^2} + \frac{X_S^4}{24k_0^4 R^4} \right) \quad (5)$$

where X_S is the east coordinates (false east is excluded) of the perspective center in the map projection frame, and k_0 is the point scale factor of the central meridian.

III. CORRECTION METHODS OF LENGTH DISTORTION

If earth curvature distortion is already accurately offset, then the objective of length correction is to “move” the light ray, which points to point G_{ec} , toward point G' . The common length correction method entails replacing the Cartesian length D in (1) with the projection length D' , and then modifying some other computational parameters to keep the equation balanced.

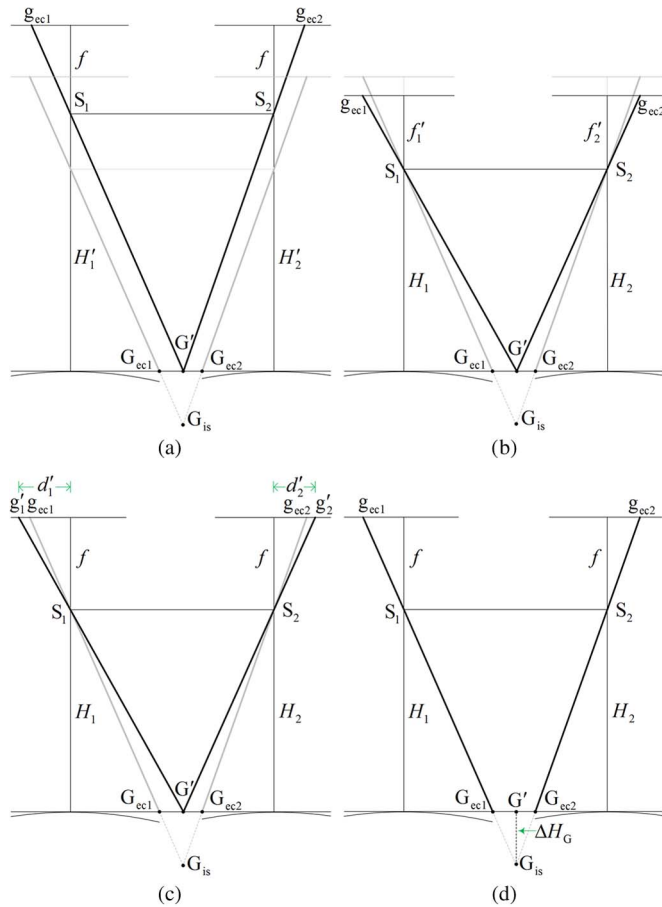


Fig. 2. Four correction methods for length distortion in map projection frame. (a) Changing flight height. (b) Changing focal length. (c) Changing image coordinates. (d) Changing object coordinates. For simplifying the illustration, the ground point is assumed to be located on the ellipsoid, and therefore the elevation of the projected ground point G' is zero. G_{ec1} and G_{ec2} are the ground positions after the earth curvature correction, which are calculated based on the left and right images, respectively. The thick gray lines, which are depicted in (a)–(c) but not explicitly in (d), represent the light rays after the earth curvature correction, and their intersection point is denoted as G_{is} . In (a)–(c), the thick black lines represent the light rays after both the earth curvature correction and the length correction; whereas in (d), the light rays are not moved because the changing object coordinates method only alters the height coordinate of the ground intersection point G_{is} .

A. Changing Flight Height

As shown in Fig. 2(a), if the flight height is chosen to be modified, then we could obtain the following equation from (1) and (4):

$$\frac{d_{ec}}{\frac{kR}{R+H_{G'}}D} = \frac{f}{H'_S - H_{G'}} \tag{6}$$

where H'_S is the flight height after corrections. Combining (1) and (6) we could get

$$H'_S = \frac{H_S - H_{G'}}{R + H_{G'}}kR + H_{G'}. \tag{7}$$

As the ground height $H_{G'}$ is usually unknown before scene restitution, the above equation could not be directly used in

engineering applications. A more practical solution is to approximately use the average terrain elevation H_{av} instead, then

$$H'_S = \frac{H_S - H_{av}}{R + H_{av}}kR + H_{av}. \tag{8}$$

Note that the average terrain elevation is always far smaller than the earth radius; therefore, $R/(R + H_{av}) \approx (R - H_{av})/R$. Then (8) could be approximately arranged to

$$H'_S = k(H_S - H_{av}) \left(1 - \frac{H_{av}}{R}\right) + H_{av}. \tag{9}$$

This equation is the length correction formula used in [3].

B. Changing Focal Length

As shown in Fig. 2(b), if the focal length is chosen to be modified, then

$$\frac{d_{ec}}{\frac{kR}{R+H_{G'}}D} = \frac{f'}{H_S - H_{G'}}. \tag{10}$$

Combining (1) and (10), and similarly, using the average terrain elevation H_{av} instead of $H_{G'}$, then we could obtain

$$f' = \frac{R + H_{av}}{kR}f. \tag{11}$$

Note that the length correction formula used in [4] is $f' = f/k$. Apparently, this equation ignores the average terrain elevation H_{av} ; therefore, its calculation accuracy is lower than (11).

C. Changing Image Coordinates

As shown in Fig. 2(c), if the image coordinates are chosen to be modified, then

$$\frac{d'}{\frac{kR}{R+H_{G'}}D} = \frac{f}{H_S - H_{G'}}. \tag{12}$$

Similarly, combining (1) and (12), and using the average terrain elevation H_{av} instead of $H_{G'}$, then we could get

$$d' = \frac{kR}{R + H_{av}}d_{ec}. \tag{13}$$

D. Changing Object Coordinates

Unlike the aforementioned three correction methods, the changing object coordinates approach does not move the light ray, but rather directly alters the height coordinates of the ground points after scene restitution. As shown in Fig. 2(a), the main effect of the changing flight height method is to vertically shift the stereoscopic image pairs and ground coordinates. If we directly add the flight height correction into the elevation coordinate of the intersection point G_{is} , it will yield a similar result, which is shown in Fig. 2(d). However, fortunately, the precise ground elevation (compared with the average terrain elevation H_{av}) is available after scene restitution, and we

therefore can accurately calculate the height correction of each ground point from (7)

$$\Delta H_G = H'_S - H_S = \frac{H_S - H_{G'}}{R + H_{G'}} kR + H_{G'} - H_S. \quad (14)$$

Note that one single ground point is usually imaged by several stations, and many height corrections therefore could be calculated by (14). As the height corrections computed from the adjacent images commonly have a similar value, the simplest solution is to average all corrections.

IV. EXPERIMENT

A. Strategy

The real aerial images are inevitably distorted by various system errors and random errors (e.g., point measurement error, atmospheric refraction), which are not easy to distinguish from map projection distortions. Therefore, this letter only uses simulated data. The experiment includes two procedures. In the first step, direct georeferencing is performed in a local space rectangular (LSR) frame. Then, we transform all ground points to the desired map projection frame. The map projection procedure employs the high-precision transverse Mercator projection formula from GeographicLib (an open source C++ library developed by Karney [12]), which has an accuracy of several nm [13]. Therefore, we could treat the ground point calculated in the LSR frame as the true value. In the second step, direct georeferencing is straightforwardly performed in the desired map projection frame. Earth curvature correction and the four different length correction methods discussed in Section III are applied to the scene restitution procedure. Finally, the computation results are compared with the true value, and the ground coordinate differences are the residuals of the map projection correction methods.

B. Data Simulation

The experimental data are divided into four groups. Data set A simulates the ideal flight and terrain conditions (i.e., all photographs are strictly vertically imaged), and the ground surface is a flat plain with a known elevation. Data set B adds random noise to the ground elevation of Data set A, and the maximum variation is 300 m. Data set C adds random noise to the EOPs of Data set A, the maximum position and attitude variation are 20 m and 3°, respectively. Data set D adds both elevation and EOPs noise to Data set A, which aims to simulate the real flight and terrain conditions. The variations are equivalent to those of value adding to Data sets B and C.

The general parameters in the four groups of experimental data include the following: the map projection frame uses a WGS84 ellipsoid and Gauss projection, with a central meridian of 117° E; the simulated images include five flight strips with a total of 25 photos, along-overlap and across-overlap are 60% and 30%, respectively; a Leica RC-30 aerial camera is simulated, its frame size is 23 cm × 23 cm, and the focal length is 153 mm; 100 ground points are evenly distributed in the cover area of a single image; and in Data set A, the geodetic coordinate of the first perspective center is (30° N, 120° E), and

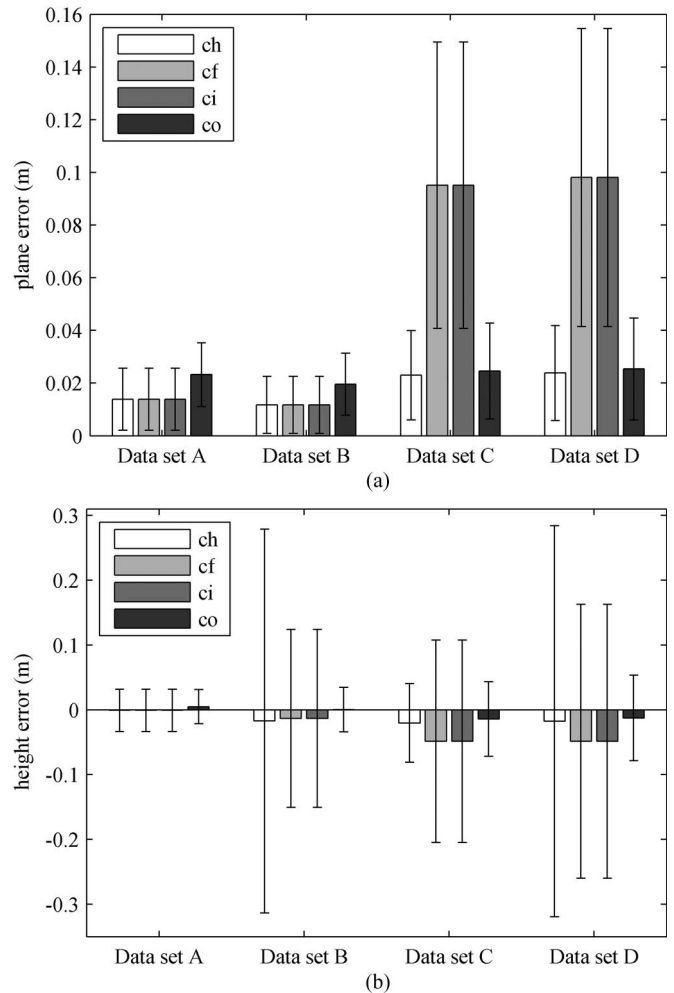


Fig. 3. Residuals of four length correction methods. (a) Plane error. (b) Height error. Abbreviations ch, cf, ci, and co represent the changing flight height method, the changing focal length method, the changing image coordinates method, and the changing object coordinates method, respectively.

the ellipsoid heights of all perspective centers and all ground points are 5000 m and 1000 m, respectively.

C. Results and Analysis

Fig. 3 shows the residuals statistics of the four length correction approaches. Comparing the results of different data sets and correction methods, we find the following:

- 1) In most experimental conditions, the changing object coordinates approach has a higher accuracy than the other three correction algorithms (i.e., the changing flight height method, the changing focal length method, and the changing image coordinates method). The major intrinsic drawback of these three methods is that the ground elevations are unknown before computing the corrections, and they therefore must approximately use the average terrain elevation instead. It can be seen from comparing the experimental results of Data sets A and B that the residual of the changing object coordinates approach is less affected by terrain fluctuation, especially for the plane coordinates.

- 2) Image inclination significantly affects the accuracy of map projection correction methods. The traditional earth curvature correction formula and all of the length correction formulas used in this letter are derived under the assumption of using the ideal vertical photography. If near vertical or oblique images are used, (1) is no longer valid, and it therefore cannot accurately correct earth curvature distortion or length distortion. Comparing the experimental results of Data sets A and C, we found that the changing flight height method and the changing object coordinates method are less affected by image inclination, while the changing focal length method and the changing image coordinates method are not.
- 3) The changing focal length method and the changing image coordinates method have the same correction effects on length distortion. By comparing (11) with (13), we found that these two correction methods are equivalent to each other (the prerequisite is to use the approximate formula of length distortion (i.e., line scale factor is replaced by point scale factor)).

V. CONCLUSION

Direct georeferencing in the map projection frame is inevitably biased by various map projection distortions, such as earth curvature distortion and length distortion. If the distortions are not accurately offset, the ground points calculated by scene restitution will be significantly deformed. This letter focused on the correction methods of length distortion only, and two new approaches (i.e., the changing image coordinates method and the changing object coordinates method) were proposed. Compared with the two existing approaches (i.e., the changing flight height method and the changing focal length method), the new approaches have many different characteristics that include the following.

Accuracy	The changing object coordinates method is less affected by terrain fluctuation and image inclination; therefore, its accuracy is usually higher than the other three methods.
Computational effort	The two existing approaches calculate one single correction for each photo, while the new approaches calculate one single correction for each image point; therefore, the two new approaches require more computational effort.

Scalability

This letter did not employ the rigorous formula of length distortion so it therefore still can be categorized as an approximate method. If higher accuracy is required, it must use a stricter map projection distortion model.

Since the corrections could be added to each image point in the two new approaches, they therefore could be extended to utilize a more complex distortion model, while the two existing approaches do not have enough scalability.

ACKNOWLEDGMENT

The authors would like to thank C. F. F. Karney for his contribution on GeographicLib. Heartfelt thanks are also given for the comments and contributions of reviewers and members of the editorial team.

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