# Camera Calibration Technique with Planar Scenes

Yongjun Zhang [1]   Zuxun Zhang [2]   Jianqing Zhang [2]

1. Center of GPS, Wuhan University, Wuhan 430079, P. R. China   yjzhang@supresoft.com.cn
2. School of Remote Sensing and Information Engineering, Wuhan University, Wuhan 430079, P. R. China

## Abstract

A flexible new camera calibration technique using 2D-DLT and bundle adjustment with planar scenes is proposed in this paper. The equation of principal line under image coordinate system represented with 2D-DLT parameters is educed using the correspondence between collinearity equations and 2D-DLT. A novel algorithm to obtain the initial value of principal point is put forward in this paper. The practical decomposition algorithm of exterior parameters using initial values of principal point, focal length and 2D-DLT parameters is discussed elaborately. Planar-scene camera calibration algorithm with bundle adjustment is addressed. For the proposed technique, either the camera or the planar pattern can be moved freely, and the motion need not be known. Very good results have been obtained with real data calibration. The calibration result can be used in some high precision applications, such as reverse engineering and industrial inspection.

**Keywords**: camera calibration, 2D-DLT, bundle adjustment, planar grid, critical motion sequences, lens distortion

## 1. Introduction

Direct Linear Transformation is developed by Abdel-Aziz (Abdel-Aziz et al, 1971). It is a well-known method used in close-range photogrammetry and other areas for its no need for initial values of camera interior and exterior parameters. Calibration of cameras is a prerequisite for the extraction of precise three-dimensional information from imagery in Photogrammetry, Computer Vision and other areas. Much work has been done in the photogrammetry community (Fang-Jenq Chen, 1997; Zhaoguang Zhu et al, 1995; Zhizhuo Wang, 1990), and also in computer vision (e.g. Tsai, 1987; Bill Triggs, 1998; P. Sturm, 1997; SongDe Ma, 1998). A large number of auto-calibration approaches have been discussed by computer vision circles, but in some cases, the result of auto-calibration can not be determined uniquely, which differs from the true value remarkably even with low noise level (Maolin Qiu, 2000).

Triggs developed a self-calibration technique from at least 5 views of a planar scene (Bill Triggs, 1998), but this technique has difficulty to initialize. Zhang put forward a camera calibration technique for planar scenes based on the orthonormal property of the rotation matrix (Zhengyou Zhang, 1998).

3D-DLT is widely used for camera calibration (e.g. Fang-Jenq Chen, 1997), but no 2D-DLT-based calibration paper has been published in the literature. This paper mainly focuses on camera calibration technique using 2D-DLT and collinearity equations with planar scenes. The proposed technique only requires the camera to view a planar pattern at a few (at least two) different orientations. We can move either the camera or the pattern freely, and the motion need not be known.

The equation of principal line represented by 2D-DLT parameters is worked out using the correspondence between collinearity equations and 2D-DLT parameters in section 2, which shows that initial value of principal point can be obtained with at least two equations of principal lines. The decomposition algorithm of exterior parameters using initial values of principal point, focal length and 2D-DLT parameters is discussed elaborately also in section 2. In section 3, planar-scene camera calibration algorithm with bundle adjustment (using collinearity equations) is addressed. Real image data are used to test the proposed technique in section 4. Very good results have been obtained, which verifies the feasibility of the proposed planar camera calibration technique. Section 5 gives some conclusions of this paper. Proof of ambiguity in camera interior parameter decomposition with single image 2D-DLT parameters is given in appendix A. In appendix B, proof of Critical Motion Sequences (CMS) for calibration is given detailedly.

## 2. 2D-DLT and Initial Values

2D-DLT can be written as (Wenhao Feng 2002)

$$x = \frac{h_1 X + h_2 Y + h_3}{h_7 X + h_8 Y + 1} \qquad y = \frac{h_4 X + h_5 Y + h_6}{h_7 X + h_8 Y + 1} \tag{1}$$

where $H = (h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8)^T$ is the 2D-DLT parameters, $X, Y$ the space point under world coordinate system (where Z=0) and $x, y$ the corresponding image point.

Given an image of model plane, the values of transformation parameters can be estimated by $AH = 0$. The solution is well known to be the eigenvector of $A^T A$ associate with the smallest eigenvalue (or equivalently, the right singular vector of $A$ associate with the smallest singular value). In order to eliminate the influence of gross errors which may be introduced by miss-match of image points and the corresponding model points, the parameters can be refined with an iterative least-square method with linearised equation (1).

The mostly used collinearity equations in photogrammetry can be written as (Zhizhuo Wang, 1990)

$$x - x_0 = -f\frac{a_1(X - X_s) + b_1(Y - Y_s) + c_1(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} \qquad y - y_0 = -f\frac{a_2(X - X_s) + b_2(Y - Y_s) + c_2(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} \tag{2}$$

where $x_0, y_0, f$ are the interior parameters, $X_S, Y_S, Z_S$ the position of camera, $X, Y, Z$ the space point under world coordinate system (Z = 0 for planar grid), $x, y$ corresponding image point and $R = \{a_i, b_i, c_i, i = 1,2,3\}$ the rotation matrix composed of three rotation angles $\phi$, $\omega$, $\kappa$

Equation (2) can be written as

$$x = \frac{\left(f\frac{a_1}{\lambda} - \frac{a_3}{\lambda}x_0\right)X + \left(f\frac{b_1}{\lambda} - \frac{b_3}{\lambda}x_0\right)Y + \left(x_0 - \frac{f}{\lambda}(a_1X_s + b_1Y_s + c_1Z_s)\right)}{-\frac{a_3}{\lambda}X - \frac{b_3}{\lambda}Y + 1}$$

$$y = \frac{\left(f\frac{a_2}{\lambda} - \frac{a_3}{\lambda}y_0\right)X + \left(f\frac{b_2}{\lambda} - \frac{b_3}{\lambda}y_0\right)Y + \left(y_0 - \frac{f}{\lambda}(a_2X_s + b_2Y_s + c_2Z_s)\right)}{-\frac{a_3}{\lambda}X - \frac{b_3}{\lambda}Y + 1} \tag{3}$$

where $\lambda = (a_3 X_s + b_3 Y_s + c_3 Z_s)$.

Comparing equation (1) with equation (3), we have

$$\begin{cases} h_1 = f\frac{a_1}{\lambda} - \frac{a_3}{\lambda}x_0 \\ h_2 = f\frac{b_1}{\lambda} - \frac{b_3}{\lambda}x_0 \end{cases} \tag{4}$$

$$\begin{cases} h_4 = f\frac{a_2}{\lambda} - \frac{a_3}{\lambda}y_0 \\ h_5 = f\frac{b_2}{\lambda} - \frac{b_3}{\lambda}y_0 \end{cases} \tag{5}$$

$$\begin{cases} h_3 = x_0 - \frac{f}{\lambda}(a_1X_s + b_1Y_s + c_1Z_s) \\ h_6 = y_0 - \frac{f}{\lambda}(a_2X_s + b_2Y_s + c_2Z_s) \end{cases} \tag{6}$$

$$\begin{cases} h_7 = -\frac{a_3}{\lambda} \\ h_8 = -\frac{b_3}{\lambda} \end{cases} \tag{7}$$

From equation (4), equation (5) and equation (7) we can obtain the following equations

$$\begin{cases} \frac{(h_1 - h_7x_0)}{f} = \frac{a_1}{\lambda} \\ \frac{(h_2 - h_8x_0)}{f} = \frac{b_1}{\lambda} \end{cases} \tag{8}$$

$$\begin{cases} \frac{(h_4 - h_7y_0)}{f} = \frac{a_2}{\lambda} \\ \frac{(h_5 - h_8y_0)}{f} = \frac{b_2}{\lambda} \end{cases} \tag{9}$$

$$\begin{cases} -h_7 = \frac{a_3}{\lambda} \\ -h_8 = \frac{b_3}{\lambda} \end{cases} \tag{10}$$

Multiplying the upper and lower parts of equation (8), equation (9) and equation (10) respectively, considering $a_1b_1 + a_2b_2 + a_3b_3 = 0$, we obtain

$$\frac{(h_1 - h_7x_0)\cdot(h_2 - h_8x_0)}{f^2} + \frac{(h_4 - h_7y_0)\cdot(h_5 - h_8y_0)}{f^2} + h_7h_8 = 0 \tag{11}$$

If the principal point ($x_0, y_0$) is known or obtained with certain approaches, the focal length can be obtained as follows

$$f = \sqrt{\frac{-(h_1 - h_7x_0)\cdot(h_2 - h_8x_0) - (h_4 - h_7y_0)\cdot(h_5 - h_8y_0)}{h_7h_8}} \tag{12}$$

Self-multiplying each items of equation (8), equation (9) and equation (10), taking $a_1^2 + a_2^2 + a_3^2 = 1$, $b_1^2 + b_2^2 + b_3^2 = 1$ into account, and canceling out $\lambda$, we obtain

$$\frac{(h_1 - h_7x_0)^2 - (h_2 - h_8x_0)^2 + (h_4 - h_7y_0)^2 - (h_5 - h_8y_0)^2}{f^2} + (h_7^2 - h_8^2) = 0 \tag{13}$$

Focal length $f$ can be canceled out using equation (11) and equation (13), then we have

$$F_h = (h_1h_8 - h_2h_7)(h_1h_7 - h_7^2 x_0 + h_2h_8 - h_8^2 x_0) + (h_4h_8 - h_5h_7)(h_4h_7 - h_7^2 y_0 + h_5h_8 - h_8^2 y_0) = 0 \quad (14)$$

In most cases, the principal point is different from the image center. There are totally 9 interior and exterior parameters $(f, x_0, y_0, \phi, \omega, \kappa, X_s, Y_s, Z_s)$ of a camera when ignore lens distortion, skew and aspect ratio, so these 9 parameters can not be decomposed uniquely from the 8 parameters of 2D-DLT. Theoretically, the principal point ($x_0, y_0$) can move freely on the principal line of image, proof is given in appendix A.

Equation (14) can also be written in the form of $(L_x \quad L_y)(x_0 \quad y_0)^T = L_c$. As we know, the principal point always lies on the principal line of image (Zhaoguang Zhu et al, 1995), so if we have at least two nonparallel principal lines, the principal point $x_0, y_0$ can be obtained by solve the over-definite linear equation $L\ X = c$.

Note that we should avoid the so-called Critical Motion Sequences (Sturm, 1997). 2D-DLT parameters among images are linearly correlated in the case of image sequences taken with a fixed camera while the planar grid is rotating around *Z*-axis. All the principal lines actually overlap each other. The principal point can not be obtained from these lines. In practice, we only need to change the orientation of the camera from one snapshot to another when the table turns around *Z*-axis. Proof is given in appendix B.

After 2D-DLT parameters, focal length and principal point are determined, the initial values of camera exterior parameters can be decomposed as follows.

Replace $\lambda$ in equation (8) and equation (9) with which derived from equation (10), we have

$$\frac{a_1}{a_3} = -\frac{(h_1 - h_7x_0)}{fh_7} \qquad \frac{b_1}{b_3} = -\frac{(h_2 - h_8x_0)}{fh_8}$$
$$\frac{a_2}{a_3} = -\frac{(h_4 - h_7y_0)}{fh_7} \qquad \frac{b_2}{b_3} = -\frac{(h_5 - h_8y_0)}{fh_8} \quad (15)$$

we obtain $b_3^2 = \dfrac{1}{1 + \dfrac{(h_2 - h_8x_0)^2}{f^2h_8^2} + \dfrac{(h_5 - h_8y_0)^2}{f^2h_8^2}}$ from equation (15) since $b_1^2 + b_2^2 + b_3^2 = 1$.

As we know, $\tan\kappa = b_1/b_2$, from equation (15) we have $\tan\kappa = b_1/b_2 = h_2 - h_8x_0 / h_5 - h_8y_0$, so $\kappa$ can be determined uniquely.

The value of $b_3$ can initially take the positive value of the square root. Compare $\kappa$ determined above with $\kappa'$ calculated from $b_1$ and $b_2$ corresponding to positive $b_3$ from equation (15). If $\kappa != \kappa'$, $b_3$ should take the negative value of the square root, then $b_1$, $b_2$ and $\omega$ can be determined from equation (15) and $\sin\omega = -b_3$ respectively.

Using the knowledge that the row vectors of rotation matrix are orthonormal, we have

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix} \quad (16)$$

we have $\tan\phi = -\dfrac{a_3}{c_3} = \dfrac{a_3}{a_1b_2 - a_2b_1} = \dfrac{1}{\dfrac{a_1}{a_3}b_2 - \dfrac{a_2}{a_3}b_1}$,

where $b_1$ and $b_2$ have been determined along with $\omega$, $a_1/a_3$ and $a_2/a_3$ are already determined in equation (15), so $\phi$ can also be determined.

The values of $\lambda$ determined from equation (8), equation (9) and equation (10) can be averaged to get the mean value, then from equation (6) and the definition of $\lambda$, we have

$$\begin{cases} h_3 = x_0 - \dfrac{f}{\lambda}(a_1X_s + b_1Y_s + c_1Z_s) \\ h_6 = y_0 - \dfrac{f}{\lambda}(a_2X_s + b_2Y_s + c_2Z_s) \\ \lambda = (a_3X_s + b_3Y_s + c_3Z_s) \end{cases} \quad (17)$$

This linear equation can be easily solved to obtain the initial values of $X_S, Y_S$ and $Z_S$. Note, as shown in above, not all the 9 elements of rotation matrix are calculated when determine $\phi, \omega$ and $\kappa$. So elements of the rotation matrix should be recalculated through $\phi, \omega$ and $\kappa$ to derive the values of $X_S, Y_S$ and $Z_S$.

## 3. Calibration with Bundle Adjustment

Bundle adjustment is the problem of refining a visual reconstruction to produce jointly optimal 3D structure and viewing parameter (camera pose and/or calibration) estimates (Bill Triggs, et al, 1999). It is widely used by photogrammetry and computer vision communities. Generally, lens distortion is larger in non-metric cameras than in metric ones, which must be determined in calibration along with the interior and exterior parameters. Skew of the two image axes will be ignored in the camera model since it is very close to 0 in most current cameras. The left part of collinearity equation (2) should consider lens distortion:

$$\begin{aligned}\Delta x &= (x - x_0)(K_1 \cdot r^2 + K_2 \cdot r^4) + \\ &P_1\left(r^2 + 2\cdot(x - x_0)^2\right) + 2\cdot P_2 (x - x_0)\cdot(y - y_0) \\ \Delta y &= (y - y_0)(K_1 \cdot r^2 + K_2 \cdot r^4) + \\ &P_2\left(r^2 + 2\cdot(y - y_0)^2\right) + 2\cdot P_1 (x - x_0)\cdot(y - y_0)\end{aligned} \tag{18}$$

where $r^2 = (x - x_0)^2 + (y - y_0)^2$, $K_1$ and $K_2$ are the first two orders of radial distortion, $P_1$ and $P_2$ are decentering distortion. $f_x$ and $f_y$ are focal length in $x$ and $y$ directions when suppose the values may be different in two directions.

Linearise equation (2) and (18), error equations of calibration can be written as

$$\begin{aligned}v_x &= \frac{\partial x}{\partial X_s}\Delta X_s + \frac{\partial x}{\partial Y_s}\Delta Y_s + \frac{\partial x}{\partial Z_s}\Delta Z_s + \frac{\partial x}{\partial \phi}\Delta\phi + \frac{\partial x}{\partial \omega}\Delta\omega + \frac{\partial x}{\partial \kappa}\Delta\kappa \\ &+ \frac{\partial x}{\partial X}\Delta X + \frac{\partial x}{\partial Y}\Delta Y + \frac{\partial x}{\partial Z}\Delta Z + \frac{\partial x}{\partial f_x}\Delta f_x + \frac{\partial x}{\partial f_y}\Delta f_y + \frac{\partial x}{\partial x_0}\Delta x_0 \\ &+ \frac{\partial x}{\partial y_0}\Delta y_0 + \frac{\partial x}{\partial K_1}\Delta K_1 + \frac{\partial x}{\partial K_2}\Delta K_2 + \frac{\partial x}{\partial P_1}\Delta P_1 + \frac{\partial x}{\partial P_2}\Delta P_2 - l_x \\ v_y &= \frac{\partial y}{\partial X_s}\Delta X_s + \frac{\partial y}{\partial Y_s}\Delta Y_s + \frac{\partial y}{\partial Z_s}\Delta Z_s + \frac{\partial y}{\partial \phi}\Delta\phi + \frac{\partial y}{\partial \omega}\Delta\omega + \frac{\partial y}{\partial \kappa}\Delta\kappa \\ &+ \frac{\partial y}{\partial X}\Delta X + \frac{\partial y}{\partial Y}\Delta Y + \frac{\partial y}{\partial Z}\Delta Z + \frac{\partial y}{\partial f_x}\Delta f_x + \frac{\partial y}{\partial f_y}\Delta f_y + \frac{\partial y}{\partial x_0}\Delta x_0 \\ &+ \frac{\partial y}{\partial y_0}\Delta y_0 + \frac{\partial y}{\partial K_1}\Delta K_1 + \frac{\partial y}{\partial K_2}\Delta K_2 + \frac{\partial y}{\partial P_1}\Delta P_1 + \frac{\partial y}{\partial P_2}\Delta P_2 - l_y\end{aligned} \tag{19}$$

After the initial values of camera parameters are determined, they can be refined with bundle adjustment. Due to the non-linear characteristics of the problem, iterations need to be performed. The corresponding items of $\Delta X$, $\Delta Y$ and $\Delta Z$ can be removed if the control points are considered to be of no errors. The status of normal equation is generally ill conditioned, so parameters should be weighted properly to ensure the stability of calibration results.

## 4. Real Data Experiment

The proposed technique is tested with real image data. The planar grid (about 45cm*45cm) rotates along with a table which turns around its vertical axis. 8 images are taken, one of them is shown in figure 1, the crosses are the match points. There are 900 designed corners in the planar grid, and the precision of each designed coordinate is about 0.3mm. It is quite easy for mechanical engineering researchers or industries to make such a grid. The image resolution of the CCD camera to be calibrated is 1300 pixels *1030pixels. Image corners are detected as the intersection of straight lines fitted to each square with precision of higher than 0.1 pixel.

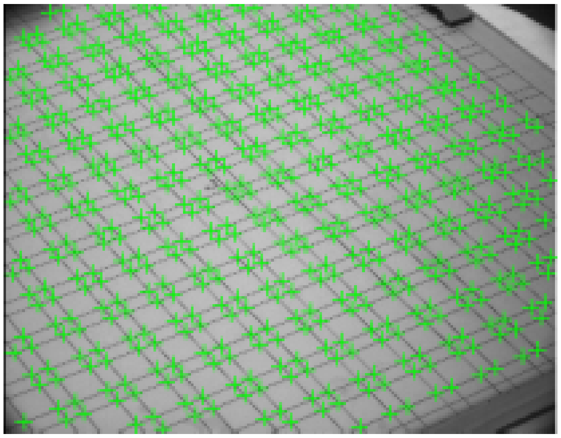

Fig.1. One of the images used for calibration

Tab.1. Estimated results of real image data (pixel)

| Items | $f_x$ | $f_y$ | $x_0$ | $y_0$ |
|---|---|---|---|---|
| Estimated | 4426.135 | 4418.137 | 652.120 | 514.730 |
| RMS | 0.201 | 0.228 | 0.080 | 0.081 |
| Items | $K_1$ | $K_2$ | $P_1$ | $P_2$ |
| Estimated | -7.416e-009 | -4.522e-015 | 6.489e-007 | 6.684e-007 |
| RMS | 1.162e-010 | 1.828e-016 | 4.654e-008 | 1.251e-008 |

There are about 500 grid points visible in each image. As we know, planar grid coordinates should be taken as weighted-unknowns in the rigorous bundle adjustment in order to eliminate the influence of imprecision of the designed grid coordinates and to get more precise camera parameters. The unit weight RMS of calibration result of the rigorous bundle adjustment is 0.08 pixel. Values and errors of interior parameters are shown in table 1.

It can be seen from table 1 that the principal point is very close to image center, and RMS errors are below 0.1 pixels. The aspect ration is 0.9982, i.e.

the pixels are nearly square. As can be calculated from the values in table 1 and equation (18), the maximum lens distortion is about 3 pixel. The deviations between detected image points and projected ones with the calibrated camera parameters and grid coordinates are also calculated. The RMSs of the deviations are about 0.1 pixels for all the 8 images, equivalent to about 0.03mm for the planar grid coordinates, which indicates the precision the proposed technique. The calibration algorithm has been successfully used in visual inspection system of industrial sheetmetal parts.

## 5. Conclusions

In this paper, we proposed a new technique for camera calibration. The proposed technique only requires the camera to observe a planar pattern at a few (at least two) different orientations. Either the camera or the pattern can be moved freely, and the motion need not be known. Compared with classical techniques that use expensive equipment such as special calibration field, the proposed technique is considerably flexible.

The proposed technique consists of two steps. In the first step, initial values are decomposed from 2D-DLT parameters. Then these initial values are refined in the second step with an iterative linear bundle adjustment using collinearity equations based on least-square criterions.

Real image data have been used to test the proposed technique, and very good results have been obtained. The proposed algorithm has been used in industrial inspection successfully.

## Appendix A: Ambiguity in Decomposition

Consider the rotation matrix composed of $A, \nu, \kappa$ where $Z$-axis is taken as the primary axis (Zhizhuo Wang, 1990), $\tan\kappa = c_1/c_2$, where $\kappa$ is the angle between principal line and y-axis. Substituting the corresponding items of equation (16) into $\tan\kappa = c_1/c_2$ results in

$$\tan\kappa = \frac{c_1}{c_2} = \frac{a_2 b_3 - a_3 b_2}{a_3 b_1 - a_1 b_3} = \frac{a_2/a_3 - b_2/b_3}{b_1/b_3 - a_1/a_3}$$

Substituting the corresponding items of equation (15) in the above equation results in

$$\tan\kappa = -(h_4 h_8 - h_5 h_7)/(h_1 h_8 - h_2 h_7)$$

Equation (14) can be represented in the form of a line $y_0 = Ax_0 + C$, where $A = tg\alpha = -\dfrac{h_1 h_8 - h_2 h_7}{h_4 h_8 - h_5 h_7}$ is the slope of the line and $\alpha$ is the angle between the line and x-axis. Obviously, $\tan\kappa = 1/\tan\alpha$, which means $\kappa = 90^0 - \alpha$, i.e. equation (14) is actually the equation of principal line in image. Clearly, as long as the principal point locates on the principal line, each group of decomposition is valid for the perspective relationship, i.e. the camera parameters can not be decomposed uniquely from single image. Mathematically, it is impossible to calibrate a camera completely from single image of a planar pattern without any other information.

## Appendix B: Proof of CMS with Fixed Camera and Turntable

Under the pinhole model, projection relationship between image and planar pattern can be written as (Zhengyou Zhang, 1998):

$$s\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = A(r_1 \quad r_2 \quad t)\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} \tag{a}$$

where $A$ is called the camera interior matrix, $r_1, r_2$ and $t$ are the first two columns of rotation matrix and camera translation respectively. $H = A(r_1 \quad r_2 \quad t)$ is called Homography between the model plane and image. It is obvious that equation (a) is actually the equation of 2D-DLT when scale $s$ is canceled out with the third row. So 2D-DLT parameters are equivalent to homography matrix while 1 is taken as the ninth element. So we take the form of homography for 2D-DLT parameters. two rotation matrix are related by

$$R_2 = R_1\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{b}$$

where $\theta$ is the angle of the relative rotation. We will use superscript $^{(1)}$ and $^{(2)}$ to denote vectors related to image 1 and 2, respectively. Substitute equation (b) into $H = A(r_1 \quad r_2 \quad t)$ results in

$$\begin{pmatrix} h_1^{(2)} \\ h_4^{(2)} \\ h_7^{(2)} \end{pmatrix} = \begin{pmatrix} h_1^{(1)} \\ h_4^{(1)} \\ h_7^{(1)} \end{pmatrix}\cos\theta + \begin{pmatrix} h_2^{(1)} \\ h_5^{(1)} \\ h_8^{(1)} \end{pmatrix}\sin\theta$$

$$\begin{pmatrix} h_2^{(2)} \\ h_5^{(2)} \\ h_8^{(2)} \end{pmatrix} = -\begin{pmatrix} h_1^{(1)} \\ h_4^{(1)} \\ h_7^{(1)} \end{pmatrix} \sin\theta + \begin{pmatrix} h_2^{(1)} \\ h_5^{(1)} \\ h_8^{(1)} \end{pmatrix} \cos\theta \qquad \text{(c)}$$

As we know, the principal line in image can be represented in the form of $y_0 = Ax_0 + C$, and the slope of this line can be written as follows

$$\tan\alpha = -(h_1 h_8 - h_2 h_7)/(h_4 h_8 - h_5 h_7) \qquad \text{(d)}$$

where $\alpha$ is the angle between the principal line and x-axis.

Substitute corresponding items of equation (c) in equation (d) result in

$$\begin{aligned} & h_1^{(2)} h_8^{(2)} - h_2^{(2)} h_7^{(2)} = \\ & \left(h_1^{(1)} \cos\theta + h_2^{(1)} \sin\theta\right)\left(-h_7^{(1)} \sin\theta + h_8^{(1)} \cos\theta\right) \\ & \quad - \left(-h_1^{(1)} \sin\theta + h_2^{(1)} \cos\theta\right)\left(h_7^{(1)} \cos\theta + h_8^{(1)} \sin\theta\right) \\ & = h_1^{(1)} h_8^{(1)} - h_2^{(1)} h_7^{(1)} \end{aligned} \qquad \text{(e)}$$

we have $h_4^{(2)} h_8^{(2)} - h_5^{(2)} h_7^{(2)} = h_4^{(1)} h_8^{(1)} - h_5^{(1)} h_7^{(1)}$ Similarly. Clearly, two principal lines are mutually parallel to each other. Further more, C can be written as follows from equation (14)

$$C = \frac{(h_1 h_8 - h_2 h_7)(h_1 h_7 + h_2 h_8) + (h_4 h_8 - h_5 h_7)(h_4 h_7 + h_5 h_8)}{h_7^2 + h_8^2} \qquad \text{(f)}$$

From equation (c) we can obtain $C^{(2)} = C^{(1)}$ without difficulty. So it is obvious that the two principal lines are actually overlapped lines under image coordinate system, which indicates that principal point can not be obtained from these overlapped lines. In practice, we only need to change the orientation of the camera or the model plane from one snapshot to another to avoid critical motion sequences.

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