Errors Analysising on Combined GPS/GLONASS Positioning

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Abstract: This paper focuses on the major errors and their reduction approaches of combined GPS/GLONASS positioning. To determine the difference in the time reference systems, different receiver clock offsets are introduced with respect to GPS and GLONASS system time. A more desirable method of introducing a fifth receiver independent unknown parameter, which can be canceled out when forming difference measurements, is discussed. The error of orbit integration and the error of transformation parameters are addressed in detail. Results of numerical integration are given. To deal with the influence of ionospheric delay, a method of forming dual-frequency ionospheric free carrier phase measurements is detailed.

Key Words: GPS/GLONASS; Difference of system time; Orbit integration; Coordinate transformation; Ionospheric correction

1. Introduction

The potential 48-satellite constellation offered by the combination of observations from both the Global Positioning System (GPS) and the Global Navigation Satellite System (GLONASS) has created considerable interest among existing GPS users and communities all over the world. The satellite's constellation and the signal in space of both GLONASS and GPS are comparable. It is well known that the GPS/GLONASS combination has better characteristics in terms of availability, accuracy, integrity, and so on.

However, the combined use of these satellite systems raises problems that must be considered. The GLONASS system time and reference frame are different from that of GPS. Moreover, to distinguish among individual satellites, GLONASS satellites employ different frequencies to broadcast their navigational information, which make existing GPS data processing software unable to process GLONASS observations.

A number of works have been done considering the forenamed problems. The system time and reference frame differences between GLONASS and GPS have been addressed by Bykhanov [1]. Approaches and the corresponding precision on GLONASS broadcast orbit computation and transformation of WGS84/PZ-90 have been discussed (Misra [2]). Four different solutions of double difference carrier phase measurements have been discussed by Leick [3]. Methods of processing GLONASS and GLONASS/GPS observations have been discussed by Habrich [4].

This paper mainly analyze the major errors and discuss their reduction approaches with respect to combined GPS/GLONASS positioning, such as the difference in the time reference systems, the errors of orbit integration and coordinate transformation, the influence of ionospheric delay and the approach of reduction. Errors related to GPS only positioning or very similar to that of GPS in combined GPS/GLONASS positioning, for instance, tropospheric delay, multipath error, Earth rotation and Earth tide errors are not discussed here.

2. Resolving the Difference of System Time

In combined GPS/GLONASS data processing, the difference between the two system times must be accounted for. Otherwise, systematic errors are introduced which will affect the combined positioning solution. To determine this difference in the time reference systems, a number of procedures are possible, and two of them will be discussed in the following.

2.1 Introducing a Second Receiver Clock Offset

Different receiver clock offsets are introduced with respect to GPS and GLONASS system time. These two clock offsets are instantaneously determined at each observation epoch together with the three unknowns of the receiver position.

The simplified non-linear observation equation for a pseudorange observation to satellite S of an arbitrary system (GPS or GLONASS) at an observer R can be written as:

$$PR_R^S = \rho_R^S + c \cdot \delta t_R - c \cdot \delta t^S \qquad (1)$$

Project supported by Dynamic Geodesy Openning Laboratry of Wuhan Institute of Geodesy and Geophisics, Chinese Academy of Science L99-(04)
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Expanded with Taylor series around an approximate position P_0, we obtain the linearized equation

$$PR_R^S = \rho_0^S + \frac{x_0 - x^S}{\rho_0^S}\cdot(x_R - x_0) + \frac{y_0 - y^S}{\rho_0^S}\cdot(y_R - y_0) + \frac{z_0 - z^S}{\rho_0^S}\cdot(z_R - z_0) + c\cdot\delta t_R - c\cdot\delta t^S \quad (2)$$

where PR_R^S is observed pseudorange; c is speed of light; x_0, y_0, z_0 are the approximate coordinates of receiver; x_R, y_R, z_R are the receiver's true coordinates to be determined; x^S, y^S, z^S are coordinates of satellite; δt_R is receiver clock offset with respect to system time; δt^S is satellite clock offset with respect to system time and $\rho_0^S = \sqrt{(x_0 - x^S)^2 + (y_0 - y^S)^2 + (z_0 - z^S)^2}$ is the geometric distance between the approximate position and the satellite position.

With the receiver clock error $\delta t_R = t_R - t_{Sys}$ (t_{Sys} being the GPS or GLONASS system time, respectively) as one of the unknowns, it is clear that in combined GPS/GLONASS processing two receiver clock offsets have to be introduced, one for the receiver offset with respect to GPS time and one for that of GLONASS time. Such that two different observation equations are obtained for a GPS satellite *i* and a GLONASS satellite *j*:

$$PR_R^{GPS\ i} = \rho_0^{GPS\ i} + \frac{x_0 - x^{GPS\ i}}{\rho_0^{GPS\ i}}\cdot(x_R - x_0) + \frac{y_0 - y^{GPS\ i}}{\rho_0^{GPS\ i}}\cdot(y_R - y_0) + \frac{z_0 - z^{GPS\ i}}{\rho_0^{GPS\ i}}\cdot(z_R - z_0) + c\cdot\delta t_{R,GPS} - c\cdot\delta t^{GPS\ i} \quad (3)$$

$$PR_R^{GLO\ j} = \rho_0^{GLO\ j} + \frac{x_0 - x^{GLO\ j}}{\rho_0^{GLO\ j}}\cdot(x_R - x_0) + \frac{y_0 - y^{GLO\ j}}{\rho_0^{GLO\ j}}\cdot(y_R - y_0) + \frac{z_0 - z^{GLO\ j}}{\rho_0^{GLO\ j}}\cdot(z_R - z_0) + c\cdot\delta t_{R,GLO} - c\cdot\delta t^{GLO\ j} \quad (4)$$

where $\delta t_{R,GPS} = t_R - t_{GPS}$ and $\delta t_{R,GLO} = t_R - t_{GLONASS}$, t_{GPS} being the system time of GPS and $t_{GLONASS}$ being the system time of GLONASS.

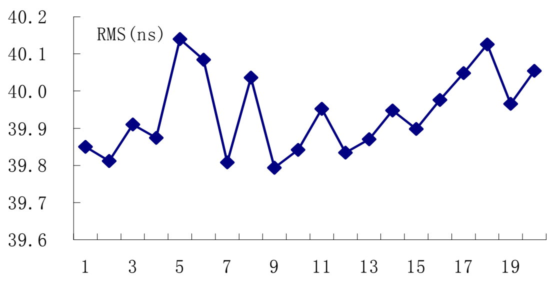

Fig.1. RMSs of time difference between GLONASS and GPS with two clock offsets

Due to the one more unknown as compared with GPS only positioning, an additional (fifth) observation is necessary to obtain a positioning solution. Since the combined use of GPS and GLONASS approximately doubles the number of observations, the sacrificing of one observation can easily be accepted. Equation (3) and (4) can be used to form the normal equation in order to resolve the five unknowns with conventional methods, such as least square adjustment or Kalman filtering.

It should be noted that a solution of these equations is only possible if there are observations of both GPS and GLONASS satellites. If all but one observed satellites are from one system, with only one satellite observation for the second system, this additional observation only contributes to the second receiver clock offset and has no influence on the computed position.

For this method, test has been performed with 20 hours real data obtained with Legacy GPS/GLONASS dual frequency receivers in Nov. 1999, see [6] for more detail. To be compared with the next algorithm, the result of GPS time offsets are subtracted by GLONASS time offsets, and the RMSs of these differences are calculated as shown in figure 1. The average of RMSs of differences is about 40.0ns, slightly higher than reported by Bykhanov [1].

2.2 Introducing the Difference of System Time

Starting with the pair of Equations (3) and (4), the receiver clock offset with respect to GLONASS system time can be rewritten as:

$$\delta t_{R,GLO} = t_R - t_{GLONASS} = t_R - t_{GPS} + t_{GPS} - t_{GLONASS} \quad (5)$$

Equation (4) then transforms to

$$PR_R^{GLO\ j} = \rho_0^{GLO\ j} + \frac{x_0 - x^{GLO\ j}}{\rho_0^{GLO\ j}}\cdot(x_R - x_0) + \frac{y_0 - y^{GLO\ j}}{\rho_0^{GLO\ j}}\cdot(y_R - y_0) + \frac{z_0 - z^{GLO\ j}}{\rho_0^{GLO\ j}}\cdot(z_R - z_0) + c\cdot\delta t_{R,GPS} + c\cdot(t_{GPS} - t_{GLONASS}) - c\cdot\delta t^{GLO\ j} \quad (6)$$

Principally, this method is equivalent to the one described in the former section, but it is more desirable. The fifth unknown parameter $(t_{GPS} - t_{GLONASS})$ as the difference of system time is independent of receivers. When differences of the same kind measurements are formed between two receivers, this unknown cancels out.

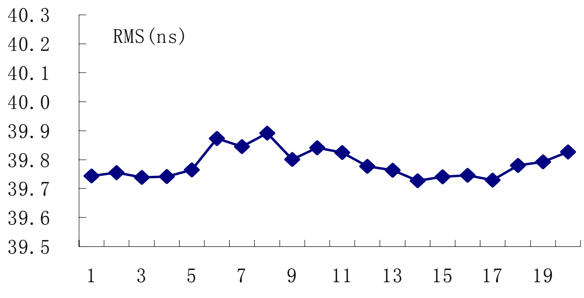

Fig.2. RMSs of time difference between GLONASS and GPS with difference of system time

Similar to the case of two separate receiver clock offsets, a solution of these equations is only possible if there are observations of both GPS and GLONASS satellites. If all but one observed satellites are from one system, with only one satellite observation for the second system, this additional observation only contributes to the difference of system time frames and has no influence on the computed position. It is reasonable in both approaches to neglect the only one observation from the second satellite system in practical data processing.

Test has also been done with the same data as the former section. The RMSs of differences of system time between GLONASS and GPS are shown in figure 2. Clearly, the average of RMSs of differences is very close to the former section, but the deviations are less than that of the former ones, which shows that introduce the difference of system time is more desirable.

3. Orbit Integration and Coordinate Transformation

Since GPS navigation and positioning have become the standard in Western countries and WGS84 is more widely utilized than PZ-90, it is considered best to transform GLONASS saellite positions from PZ-90 to WGS84 in combined navigation and positioning, thus the user position is also obtained in WGS84.

GLONASS broadcast ephemerides contain the satellite position in PZ-90 at a reference time, together with the satellite velocity and its acceleration due to luni-solar attraction. To obtain GLONASS satellite position at an epoch other than reference time, the satellite's equation of motion has to be integrated.

The error of GLONASS satellite coordinate is mainly composed of the error of orbit integration and the error of transformation parameters, which will be addressed in the following.

3.1 Numerical Integration

In compliance with Newton's laws of motion, the motion of a satellite orbiting the earth is determined by the forces acting on it. The primary force acting on satellite is that caused by Earth's gravity field potential. Expanding the non-spherical part of the gravational potential into spherical harmonics, taking the influence of earth rotation into account, assuming the acceleration of the satellite due to lunar and solar gravitation to be constant over a short time span of integration, and ignoring all other insignificant forces, the GLONASS satellite's equation of motion can be finally written as [6]:

$$\frac{dV_x}{dt} = -\frac{GM}{r^3}x + \frac{3}{2}C_{20}\frac{GMa_e^2}{r^5}x\left(1-\frac{5z^2}{r^2}\right) + x''_{LS} + \varpi^2 x + 2\varpi V_y$$

$$\frac{dV_y}{dt} = -\frac{GM}{r^3}y + \frac{3}{2}C_{20}\frac{GMa_e^2}{r^5}y\left(1-\frac{5z^2}{r^2}\right) + y''_{LS} + \varpi^2 y - 2\varpi V_x$$

$$\frac{dV_z}{dt} = -\frac{GM}{r^3}z + \frac{3}{2}C_{20}\frac{GMa_e^2}{r^5}z\left(3-\frac{5z^2}{r^2}\right) + z''_{LS} \qquad (7)$$

where x, y, z are the coordinates of satellite; $x''_{LS}, y''_{LS}, z''_{LS}$ are the luni-solar acceleration; $r = \sqrt{x^2 + y^2 + z^2}$ is distance of satellite to center of earth; a_e = 6378136 m is equatorial radius of earth; GM=$3.9860044 10^{14}$ m^3/s^2 is the gravitational constant of earth; $C_{20} = -1.08263 \cdot 10^{-3}$ is second zonal coefficient and $\omega = 7.292115 \cdot 10^{-5} s^{-1}$ is rotation rate of earth.

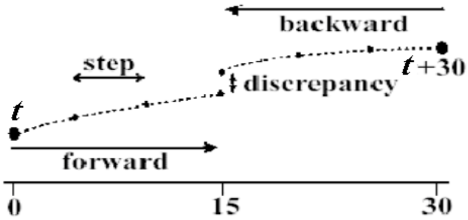

Fig 3. Discrepancy of forward and backward integration

Tab. 1. Discrepancies between forward and backward orbit integration (m)

Step	Components	Max.	Min.	Average	RMS
0.1	X	1.280	-1.004	0.005	0.812
	Y	0.698	-2.158	-0.698	0.734
	Z	1.528	-1.205	0.453	0.985
1.0	X	1.257	-1.344	0.006	0.800
	Y	0.668	-2.121	-0.681	0.744
	Z	1.549	-1.238	0.469	0.955
30.0	X	1.305	-1.044	-0.034	0.843
	Y	0.642	-2.258	-0.612	0.699
	Z	1.554	-1.253	0.368	0.915

The discrepancy between forward and backward integration of 15 min is shown in figure 3. The fourth order Runge-Kutta method is used for the numerical integration. table 1 shows the discrepancies between a forward integration (initial value is the broadcast position for epoch *i*) and a backward integration(initial value is the broadcast position for epoch *i + 30* min) for the epoch *i + 15* min, using three different integration step widths(0.1, 1 and 30 sec). The 0.1, 1 and 30 sec integration step widths lead to nearly identical results, with the root-mean-square(RMS)s no more than 1 m for all components.

3.2 Coordinate Transformation

Considering the three-dimensional transformation between PZ-90 and WGS84, the well known 7 parameter Helmert transformation can be used:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS84} = \begin{bmatrix} dX_0 \\ dY_0 \\ dZ_0 \end{bmatrix} + (1+dm)\begin{bmatrix} 1 & \beta_Z & -\beta_Y \\ -\beta_Z & 1 & \beta_X \\ \beta_Y & -\beta_X & 1 \end{bmatrix}\begin{bmatrix} U \\ V \\ W \end{bmatrix}_{PZ-90} \quad (8)$$

where dX_0, dY_0, dZ_0 are coordinates of the origin of PZ-90 in frame WGS84; $\beta_X, \beta_Y, \beta_Z$ is differential rotations around the axes (U, V, W) respectively; *dm* is differential scale change

There are several possible methods to determine the transformation parameters from PZ-90 to WGS84. Three points known in both systems are mathematically sufficient to calculate the desired parameters. However, as much points as possible are desired to obtain a good quality of the derived parameters. For the computation of these parameters, ground-based techniques and space-based techniques are used by researchers [5].

One of the most desirable parameters is given by Rossbach [5]. When applied to the station coordinates, this transformation yields a residual of 30-40 cm RMS.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS84} = \begin{bmatrix} 0.0 \\ 0.0 \\ 0.0 \end{bmatrix} + \begin{bmatrix} 1 & -1.6\times10^{-6} & 0 \\ 1.6\times10^{-6} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} U \\ V \\ W \end{bmatrix}_{PZ-90} \quad (9)$$

It is given by Bykhanov [1] that an internal coordinate system transformation accuracy of 0.3 m can be achieved. The accuracy of PZ-90 relative to WGS84 is confirmed by Earth rotation data collected in the Systems Control Center from GLONASS observations. These ERP (GL) differs from the respective ERP (IERS) by no more than 0.3 m.

The error with respect to GLONASS satellite coordinate is mainly composed of the error of orbit integration and the error of transformation parameters. So the synthetic influence of the two errors is about 1 m, which is sufficient for close range differencing positioning. Combined GPS/GLONASS precise ephemerides are available in post processing positioning from IGEX.

4. Ionospheric Correction

Similar to GPS only positioning, ionospheric delay is also one of the major constraints in combined GPS/GLONASS precise positioning. But the ionospheric delay of L_1 or L_2 signal is different for different GLONASS satellites, whereas it is identical for GPS satellites, a reduction method applicable for combined GPS/GLONASS positioning has to be developed. Since GLONASS grants full access to the L_2 frequency, it enables the properly equipped user to deal with the ionosphere problem by using dual-frequency ionospheric free carrier phase measurements.

The ionospheric path delay of a GPS or GLONASS satellite signal depends on the electron content of the ionosphere, the frequency of the signal and the distance that the signal travels through the ionosphere, which in turn depends on the satellite elevation. It can be written as:

$$c\cdot\delta t_R^{S,IONO} = \frac{1}{\cos z}\frac{40.3m^3/s^2}{f^2}TEC \quad (10)$$

where z is the zenith distance of signal at ionospheric piercing point; *f* is frequency of carrier signal; TEC is total electron content of ionosphere.

The GLONASS observation equation for carrier phase measurements from receiver R to satellite S scaled in cycles can be written as:

$$\begin{aligned}\varphi_R^S = \frac{1}{\lambda^S}\rho_R^S + N_R^S + f^S\cdot\delta t_R - f^S\cdot\delta t^S + \\ f^S\cdot\delta t_R^{S,Trop} - f^S\cdot\delta t_R^{S,Iono} + \varepsilon_R^S\end{aligned} \quad (11)$$

where $\rho_R^S = \sqrt{(x_R - x^S)^2 + (y_R - y^S)^2 + (z_R - z^S)^2}$; x_R, y_R, z_R are coordinates of receiver; x^S, y^S, z^S are coordinates of satellite; λ^S is wavelength of carrier signal of satellite S; f^S is frequency of satellite signal; φ_R^S is carrier phase measurement of receiver R to satellite S; N_R^S is carrier phase ambiguity of receiver R to satellite S; δt_R is the receiver clock offset with respect to system time; δt^S is the satellite clock offset with respect to system time; $\delta t_R^{S,Trop}$ is tropospheric delay of signal; $\delta t_R^{S,Iono}$ is ionospheric advance of signal; ε_R^S is measurement noise.

To ensure the ionospheric free linear combination of carrier phase measurements at the order of magnitude of truly measured values, this combination can then be written as:

$$\varphi_{R,IF}^S = \frac{k_1}{k_1+k_2}\cdot\varphi_{R,L_1}^S + \frac{k_2}{k_1+k_2}\cdot\varphi_{R,L_2}^S \quad (12)$$

where k_1 and k_2 are arbitrary factors to be determined in such a way that $\varphi_{R,IF}^S$ no longer contains any influence of the ionosphere.

Postulating that the ionospheric influence in equation (11) on this linear combination disappear:

$$k_1\cdot f_{L_1}\cdot\delta t_R^{S,Iono}(f_{L_1}) + k_2\cdot f_{L_2}\cdot\delta t_R^{S,Iono}(f_{L_2}) = 0 \quad (13)$$

For convenience, choose $k_1 = 1$, then obtain $k_2 = -\frac{f_{L_1}}{f_{L_2}}\cdot\frac{\delta t_R^{S,Iono}(f_{L_1})}{\delta t_R^{S,Iono}(f_{L_2})}$. By inserting equation (10) we get $k_2 = -f_{L_2}/f_{L_1}$.

So the ionospheric free linear combination of carrier phase measurements (12) can be rewritten as:

$$\varphi_{R,IF}^S = \frac{f_{L_1}}{f_{L_1}-f_{L_2}}\cdot\varphi_{R,L_1}^S - \frac{f_{L_2}}{f_{L_1}-f_{L_2}}\cdot\varphi_{R,L_2}^S \quad (14)$$

As for GLONASS satellite, the factor $k_2 = -f_{L_2}/f_{L_1} = -7/9$ is a non-integer value, the ionospheric free linear combination ambiguity $N_{R,IF}^{S} = \frac{f_{L_1}}{f_{L_1}-f_{L_2}} \cdot N_{R,L_1}^{S} - \frac{f_{L_2}}{f_{L_1}-f_{L_2}} \cdot N_{R,L_2}^{S} = \frac{1}{1-f_{L_2}/f_{L_1}} \cdot \left(N_{R,L_1}^{S} - \frac{f_{L_2}}{f_{L_1}} \cdot N_{R,L_2}^{S} \right)$ is no longer an integer value. To retain the integer nature of this value, sometimes the so-called L_0 combination $\varphi_{L_0}^{S} = 9 \cdot \varphi_{R,L_1}^{S} - 7 \cdot \varphi_{R,L_2}^{S}$ can be used as the ionospheric free linear combination. However, under the assumption that the measurement noise of L_1 carrier phase is identical to that of L_2 carrier phase, $\sigma_{\varphi_{L_1}} = \sigma_{\varphi_{L_2}} = \sigma_{\varphi}$, the noise of this combination is $\sigma_{\varphi_{L_0}} = \sqrt{9^2+7^2} \cdot \sigma_{\varphi} \approx 11.4 \cdot \sigma_{\varphi}$ The wavelength λ_{L_0} of this combination is

$$\lambda_{L_0} = \frac{c}{9 \cdot f_{L_1} - 7 \cdot f_{L_2}} = \frac{c}{f_{L_1}} \cdot \frac{1}{9 - 7 \cdot f_{L_2}/f_{L_1}} = \lambda_{L_1} \cdot \frac{9}{9^2 - 7^2} = 0.28125 \cdot \lambda_{L_1} \quad (15)$$

This is about 5.26 cm for GLONASS frequency number 1.

For GPS, $k_2 = -f_{L_2}/f_{L_1} = -60/77$, the L_0 combination noise is $\sigma_{\varphi_{L_0}} = \sqrt{77^2+60^2} \cdot \sigma_{\varphi} \approx 97.6 \cdot \sigma_{\varphi}$ at a wavelength of $\lambda_{L_0} = \frac{c}{77 \cdot f_{L_1} - 60 \cdot f_{L_2}} \approx 0.033\, \lambda_{L_1}$, which corresponds to approximately 0.6 cm.

High noise and small wavelength have precluded this combination from having any significant importance for GPS carrier phase positioning. But for GLONASS, these values are much more favorable. Today's GLONASS receivers provide a noise level around 0.5-1 mm (1 sigma) for carrier phase measurements [5]. This would mean a noise level of 0.57-1.14 cm for the L_0 combination, well below the 5.26 cm wavelength of the L_0 signal.

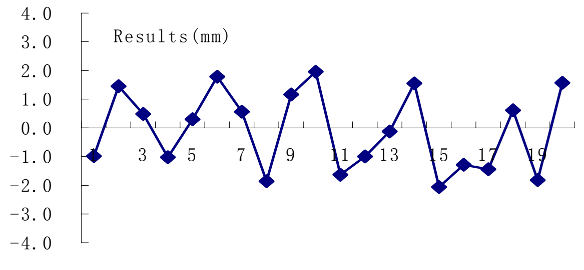

Fig. 4. Deviations of baseline length

20 hours' real data are used to test the performance of the ionospheric free linear combination. The ionospheric free observations of GPS are created with the standard algorithm used by most software such as GAMIT or BERNESE, observations of GLONASS are created with forenamed algorithm. 20 hours' real data are subdivided to 20 segments, each 1 hour's. The baseline length is calculated and compared with the "true length" obtained with the 20 hours data. Deviations of each hour's baseline length are shown in figure 4.

It can be seen that the differences between 1 hours' results and the "true length" are all below 4 mm, and the RMS of these differences is about 2mm, higher than the results of [6].

5. Conclusion

In combined GPS/GLONASS data processing, the differences between the system times must be accounted for. Two procedures to reduce the error of time reference differences between GPS and GLONASS are discussed. As can be seen from the real data tests, the method of introducing the fifth unknown as the difference of system time is more desirable. And the characteristic of the unknown is independent of receivers. When differences of the same kind_measurements are formed between two receivers, this unknown cancels out.

The error with respect to GLONASS satellite coordinate is mainly composed of the error of orbit integration and the error of coordinate transformation. As shown in above, the synthetic influence of the two errors is about 1 m (1 sigma), which is sufficient for close range differencing navigation and positioning.

The ionospheric free linear combination ambiguity $N_{R,IF}^{S}$ is not an integer value, the L_0 combination can be formed to retain the integer nature of ambiguity. High noise and small wavelength have no practical importance for GPS carrier phase positioning, whereas it is much more favorable for GLONASS. Results of real data show the effectivity of L_0 combination for GLONASS observations

References

1. Bykhanov E., Timing and Positioning with GLONASS and GPS, GPS Solutions, Vol. 3, No. 1, pp. 26-31, 1999
2. Misra P., Integrated Use of GPS and GLONASS: Transformation between WGS84 and PZ-90, Proceedings of ION GPS-96, September 17-20, Kansas City, MO, USA, 261-270, 1996
3. Leick A., Aspects of GLONASS Carrier Phase Differencing, GPS Solutions, Vol. 2, No. 1, pp. 36-41,1998
4. Habrich H., Experiences of the BKG in Processing GLONASS and Combined GLONASS/GPS Observations, D 60598 Frankfurt Main, Germany, 1998
5. Rossbach U., Positioning and Navigation Using the Russian Satellite System GLONASS, Doctoral Dissertation, University of Munich, Germany, 2000
6. Zhang Y J, Research of Data Processing Software on Combined GPS/GLONASS Relative Positioning, Master's Thesis，Wuhan University，China, 2000 (in Chinese)