

Combined GPS/ GLONASS Data Processing

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ABSTRACT To obtain the GLONASS satellite position at an epoch other than reference time, the satellite's equation of motion has to be integrated with broadcasting ephemerides. The iterative detecting and repairing method of cycle slips based on triple difference residuals for combined GPS/ GLONASS positioning and the iterative ambiguity resolution approach suitable for combined post processing positioning are discussed systematically. Experiments show that millimeter accuracy can be achieved in short baselines with a few hours' dual frequency or even single frequency GPS/ GLONASS carrier phase observations, and the precision of dual frequency observations is distinctly higher than that of single frequency observations.

1 Introduction

The system time and reference frame of GLONASS are different from those of GPS. Moreover, to distinguish among individual satellites, GLONASS satellites employ different frequencies to broadcast their navigational information, which make the existing GPS data processing software unable to process GLONASS observations. Considering these problems, a number of proposals have been put forward. Four different solutions of double difference carrier phase measurements were discussed^[1]. A common frequency approach for combined GPS/ GLONASS ambiguity resolution was addressed by Rossbach in his doctoral dissertation in detail^[2].

This paper is mainly focused on combined GPS/ GLONASS data processing. Positions of GLONASS satellites are interpolated according to the formulas of GLONASS ICD^[3] within an interval of 15 minutes. The time system of GLONASS is unified to that of GPS, and the GLONASS reference frame PZ-90 is also transformed to GPS reference frame WGS-84. The iterative detecting and repairing method of cycle slips and the iterative ambiguity resolution approach for post processing positioning are discussed. Some experimental results are given.

2 Numerical integration of GLONASS ephemerides

GLONASS broadcast ephemerides contain the satellite position in PZ-90 at a reference time, together with the satellite velocity and its acceleration due to luni-solar attraction. These data are updated every 30 minutes. To obtain satellite position at an epoch other than the reference time, the satellite's equation

of motion has to be integrated by using the given values in ephemerides as initial values.

According to Newton’s laws of motion ,the motion of a satellite orbiting the earth is determined by the forces acting on it. The primary force acting on the satellite is caused by Earth’s gravity field potential. Expanding the non-spherical part of the

$$\begin{aligned} \frac{dV_x}{dt} &= -\frac{GM}{r^3}x + \frac{3}{2}C_{20}\frac{GMa_e^2}{r^5}x\left(1 - \frac{5z^2}{r^2}\right) + x_{LS} + \tilde{~}^2x + 2\tilde{~}V_y \\ \frac{dV_y}{dt} &= -\frac{GM}{r^3}y + \frac{3}{2}C_{20}\frac{GMa_e^2}{r^5}y\left(1 - \frac{5z^2}{r^2}\right) + y_{LS} + \tilde{~}^2y - 2\tilde{~}V_x \\ \frac{dV_z}{dt} &= -\frac{GM}{r^3}z + \frac{3}{2}C_{20}\frac{GMa_e^2}{r^5}z\left(1 - \frac{5z^2}{r^2}\right) + z_{LS} \end{aligned} \tag{1}$$

where x, y, z are the satellite’s coordinates; x_{LS}, y_{LS}, z_{LS} are the luni-solar acceleration; $r = \sqrt{x^2 + y^2 + z^2}$ is the distance from the satellite to the center of Earth; $a_e = 6\,378\,136$ m is the Earth’s equatorial radius; $GM = 3.986\,004\,4 \times 10^{14}$ m³/s² is the Earth’s gravitational constant; $C_{20} = -1.082\,63 \times 10^{-3}$ is the second zonal coefficient; $\tilde{~} = 7.292\,115 \times 10^{-5}$ s⁻¹ is the Earth’s rotation rate.

The fourth order Runge-Kutta method can be used for the necessary numerical integration ,and 30 seconds can be adopted as step-length^[4].

3 Cycle slips detection

Generally, double difference model is used in combined GPS/ GLONASS relative positioning. Considering that the GLONASS satellites employ the different frequencies to broadcast their navigational information ,triple difference residuals can be used to detect and correct the cycle slips.

Triple difference residuals can be interpreted as :

$$r_{kl}^{ij}(t_2 - t_1) = b_i^i - b_j^j \tag{2}$$

where $b_i = N_{kl}^i(t_2) - N_{kl}^i(t_1)$, $b_j = N_{kl}^j(t_2) - N_{kl}^j(t_1)$; i, j are the wave lengths of satellites i and j ; k, l are the two receiver stations; t_1, t_2 are the two epochs; N_{kl}^i, N_{kl}^j are the ambiguities of satellites i and j .

Eq. (2) leads to the following cases: if no cycle slip occurs ,residuals should be close to zero ,the difference $b_i - b_j$ can not be determined because $i - j = 0$, and as soon as b_i or b_j is known ,the

gravitational potential into spherical harmonics ,taking into account the influence of Earth rotation ,assuming the acceleration of the satellite due to lunar and solar gravitation to be constant over a short time span of integration ,and ignoring all other insignificant forces ,the satellite’s equation of motion can be finally written as^[4]

cycle slip can be assigned to a single satellite.

In order to calculate the integer numbers of cycles for the single difference observation equations ,the triple difference residuals have to be converted into units of cycles. The conversion is shown in the following formula :

$$\frac{r_{kl}^{ij}(t_2 - t_1)}{i} = b_i - b_j + b_j \frac{1}{i} \tag{3}$$

The integer numbers are destroyed by a bias term. Besides the wavelength difference ,the bias term depends on the size of the slip. Obviously ,the bias term is smallest for the smallest wavelength difference between the two satellites. This leads to an iterative approach for cycle slip correction. A scheme for this algorithm is shown below.

- 1) Compute triple difference residuals for all satellite pairs and select pairs with “no slip”,in other words ,select the pairs with residuals smaller than a certain value set in advance.
- 2) Find satellite pair with the smallest residual in all satellites with “no slip” and set $b_i = b_j = 0$.
- 3) Find the satellite pair “with slip” which includes one of the “clean” satellites and assign the cycle slip to another satellite.
- 4) Correct cycle slip to the single difference observation and start next iteration.
- 5) Introduce new ambiguities for cycle slips that can not be assigned to a single satellite.

4 Ambiguity resolution approach

The different carrier frequencies of GLONASS

satellites do not cause ambiguities' non-integer problems for zero difference and single difference observations, however, this is not the case in double difference observations between GLONASS satellites or between GLONASS and GPS satellites. This problem must be resolved in precisely positioning. An applicable ambiguity resolution approach to post processing will be discussed below^[4,5].

The zero difference observations (raw observations) can be written as

$$r_k^i = c \cdot t_k^i + N_k^i \cdot \lambda^i + c \cdot t_k - c \cdot t^i \tag{4}$$

where c is the speed of light; t_k^i is the signal's travelling time between satellite i and receiver k ; N_k^i is the unknown integer number of cycles (Ambiguity); λ^i is the nominal wavelength of signal from satellite i ; t^i is the satellite clock error at time of emission; t_k is the receiver clock error at time of reception.

Single difference observations can be written as

$$r_{kl}^i = c \cdot t_{kl}^i + N_{kl}^i \cdot \lambda^i + c \cdot t_{kl} \tag{5}$$

where

$$\begin{aligned} t_{kl}^i &= t_k^i - t_l^i \\ N_{kl}^i &= N_k^i - N_l^i \\ t_{kl} &= t_k - t_l \end{aligned}$$

And the double difference observations can be written as

$$r_{kl}^{ij} = c \cdot t_{kl}^{ij} + N_{kl}^{ij} \cdot \lambda^i + N_{kl}^j \cdot \lambda^j \tag{6}$$

where

$$\begin{aligned} t_{kl}^{ij} &= t_{kl}^i - t_{kl}^j \\ N_{kl}^{ij} &= N_{kl}^i - N_{kl}^j \end{aligned}$$

In fact, the initial double difference observations do not have the form shown in Eq. (6). A transformation has been done about the ambiguity terms, which is shown in Eq. (7). So every double difference equation contains an integer-characterized double difference ambiguity and a single difference ambiguity related to part of the reference satellite.

$$\begin{aligned} N_{kl}^i \cdot \lambda^i - N_{kl}^j \cdot \lambda^j &= \\ N_{kl}^i \cdot \lambda^i - N_{kl}^j \cdot \lambda^i + N_{kl}^j \cdot \lambda^i - N_{kl}^j \cdot \lambda^j &= \\ (N_{kl}^i - N_{kl}^j) \cdot \lambda^i + N_{kl}^j (\lambda^i - \lambda^j) &= \\ N_{kl}^{ij} \cdot \lambda^i + N_{kl}^j \cdot \lambda^{ij} & \tag{7} \end{aligned}$$

The bias term $b^{ij} = N_{kl}^j \cdot \lambda^{ij}$ depends on the wavelength differences of the two satellites and the single difference ambiguity of the reference

satellite. In order to fix the double difference ambiguities, the single difference ambiguities of the reference satellite have to be known. In the case of small wavelength differences of the two satellites or the single difference ambiguities known with an accuracy of a few cycles, the bias term is smaller than 0.1 cycle, and the double difference ambiguities may be found as integers.

An iterative approach can be used to fix the double difference ambiguities. The ambiguities of satellite pairs with small wavelength differences are solved first. The following iterations show significant smaller RMS errors for the single difference ambiguities and the double difference ambiguities may be obtained for satellite pairs with larger wavelength difference.

1) The normal equation system for single difference phase observations is set up. There are n single difference equations for observations of n satellites.

2) These equations are enhanced by a priori constraint or by code observations to remove the singularity of the system, and the single difference ambiguities are estimated as real values.

3) All possible double difference ambiguities N_{kl}^{ij} are computed, and their formal errors are estimated from the covariance matrix Q of the single difference ambiguities. The formal errors are highly correlated with the difference in the wavelengths of the satellites involved in the double difference. The smaller the difference of wavelengths is, the smaller the formal error is.

4) The double difference combination with the smallest formal error is fixed to an integer number.

5) One of the two single difference ambiguities involved in forming the double difference combination may be eliminated from the normal equation system. And the next iteration of the iterative ambiguity resolution approach begins.

6) The unresolved single difference ambiguity is fixed to an integer in the final solution.

5 Experiments

An example of positioning using GPS and GLONASS double difference carrier phases is

shown in Fig. 1 and Table 1. The positions are computed from data logged by two GG-24 GPS/GLONASS single frequency receivers ,with one as reference station and the other as user station. The baseline is about 50 meters long. There are at least six GPS satellites and two GLONASS satellites in view during the successive observation time of 30 hours. Raw data are divided into segments with two hours and 6 hours long respectively. Each seg-

ment contains GPS only and GPS/ GLONASS data. The GPS only data are processed with BERNESE GPS software , and the GPS/ GLONASS observations are processed according to the forenamed theory, which is notated as "iterative approach" below. All the comparisons are based on baseline vectors because of the unknown coordinates of the two stations.

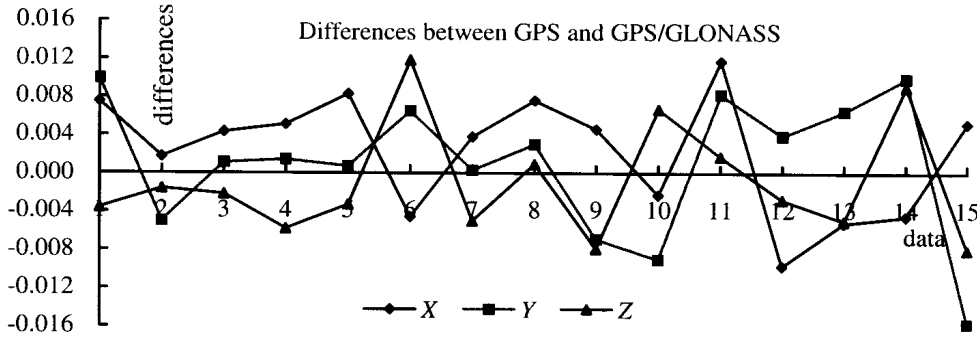


Fig. 1 Comparison between two hours ' GPS only and GPS/ GLONASS single frequency data/ m

The statistics of GPS only and GPS/ GLONASS results are shown in Fig. 1 and Table 1. The maximum discrepancy between two hours ' GPS only and GPS/ GLONASS results is about 1. 6 centimeters and the RMS errors of all components are no more than eight millimeters. The precision of six hours '

data results is much higher than two hours ' as assumed ,with the RMS errors of all components less than three millimeters. It is shown from Fig. 1 and Table 1 that the results by iterative approach with GPS/ GLONASS data have not significant system bias compared with the results of GPS only data.

Table 1 Results statistics of GPS and corresponding GPS/ GLONASS single frequency data/ m

| Statistic items | Two hours | | | Six hours | | |
|-----------------|-----------|-----------|-----------|-----------|-----------|-----------|
| | X | Y | Z | X | Y | Z |
| RMS | 0.005 8 | 0.007 4 | 0.006 0 | 0.002 9 | 0.001 6 | 0.002 4 |
| Max. | 0.011 7 | 0.009 9 | 0.011 8 | 0.005 3 | 0.002 9 | 0.001 9 |
| Min. | - 0.009 7 | - 0.015 7 | - 0.008 0 | - 0.001 5 | - 0.001 2 | - 0.004 0 |
| Average | 0.002 2 | 0.001 0 | - 0.001 0 | 0.002 2 | 0.001 0 | - 0.001 0 |

The segments of two hours ' GPS/ GLONASS data are also processed with Pinnacle software developed by JAVAD Positioning Systems , and the results are compared with the results already processed with iterative approach. Table 2 shows that the RMS errors of all components are about six millimeters and the maximum difference is less than two centimeters. It is shown that the result of iterative approach has not significant system bias compared with the result of Pinnacle software with the same GPS/ GLONASS data.

Another experiment was made with two JAVAD GPS/ GLONASS dual frequency receivers , with one

as a reference station and the other as a user station. The data of 40 hours ' successive observation are divided into 20 segments ,two hours each , and the results by iterative approach are compared with the results of Pinnacle , which are shown in Fig. 2 and Table 3 . All the comparisons are based on

Table 2 Comparison between iterative approach and pinnacle of GPS/ GLONASS single frequency data/ m

| Statistic items | X | Y | Z |
|-----------------|-----------|-----------|-----------|
| RMS | 0.006 6 | 0.006 3 | 0.006 3 |
| Max. | - 0.000 2 | 0.006 1 | 0.011 7 |
| Min. | - 0.018 2 | - 0.014 2 | - 0.005 0 |
| Average | - 0.004 4 | - 0.002 6 | 0.003 4 |

baseline vectors because of the unknown coordinates of the two stations. It is obvious that the precision of two hours' dual frequency observations is

only a few millimeters, higher than the precision of the single frequency observations in the same length of time.

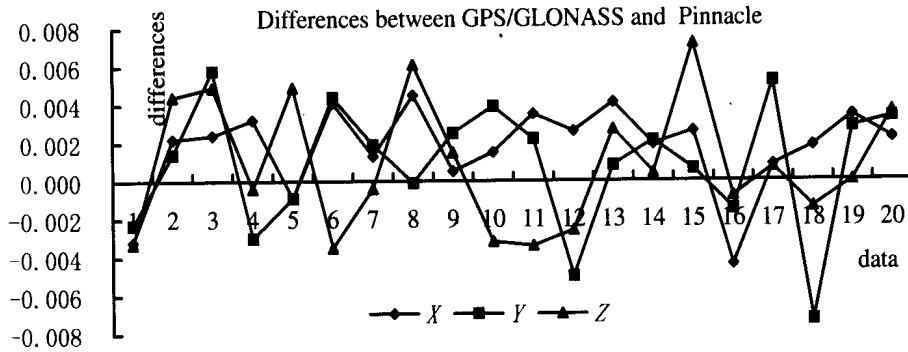


Fig. 2 Comparison between iterative approach and pinnacle of 2 hours' GPS/ GLONASS dual frequency data/ m

Table 3 Comparison between iterative approach and pinnacle of GPS/ GLONASS dual frequency data/ m

| Statistic items | X | Y | Z |
|-----------------|-----------|-----------|-----------|
| RMS | 0.002 3 | 0.003 4 | 0.003 4 |
| Max. | 0.004 5 | 0.005 8 | 0.007 2 |
| Min. | - 0.004 4 | - 0.007 3 | - 0.003 5 |
| Average | 0.001 7 | 0.000 8 | 0.000 9 |

It is reasonable that in post processing combined GPS/ GLONASS carrier phase positioning, millimeter accuracy can be achieved in short baselines with a few hours' dual frequency or even single frequency observations, and the result precision of dual frequency observations is significantly higher than that of single frequency observations.

6 Conclusions

Step length of 30 seconds can be adopted for fourth order Runge-kutta integration with the broadcast ephemerides updated every 30 minutes. In short baseline positioning millimeter accuracy can be achieved in post processing with a few hours' dual frequency or even single frequency GPS/ GLONASS carrier phase observations, and the result precision of dual frequency observations is dis-

tinctly higher than that of single frequency observations.

The results of iterative approach have not significant system bias when compared with the results of BERNESE or the results of Pinnacle software, which prove the correctness of the iterative detecting and repairing method of cycle slips based on triple difference residuals and the iterative ambiguity resolution approach applicable for combined positioning.

References

- 1 Leick A, et al. (1998) Aspects of GLONASS carrier phase differencing. *GPS Solutions*, 2(1) :36-41
- 2 Rossbach U (2000) Positioning and navigation using the russian satellite system GLONASS: [Ph.D Dissertation]. Germany:University of Munich.
- 3 Coordination Scientific Information Center (1995) GLONASS interface control document. Moscow, Russia.
- 4 Zhang YJ (2000) Research of data processing software on combined GPS/ GLONASS relative positioning: [Master's Thesis]. Wuhan:Wuhan University.
- 5 Habrich H (1998) Experiences of the BKG in processing GLONASS and combined GLONASS/ GPS observations. Frankfurt, Germany.
- 6 Stewart M, Tsakiri M (1998) GLONASS broadcast orbit computation. *GPS Solutions*, 2(2) :16-27